Fundamentals of Dynamic Frequency Hopping in Cellular Systems

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***Abstract*—We examine techniques for increasing spectral efficiency of cellular systems by using slow frequency hopping (FH) with dynamic frequency-hop (DFH) pattern adaptation. We first present analytical results illustrating the improvements in frequency outage probabilities obtained by DFH in comparison with random frequency hopping (RFH). Next, we show simulation results comparing the performance of various DFH and RFH techniques. System performance is expressed by cumulative distribution functions of codeword error rates. Systems that we study incorporate channel coding, interleaving, antenna diversity, and power control. Analysis and simulations consider the effects of path loss, shadowing, Rayleigh fading, cochannel interference, coherence bandwidth, voice activity, and occupancy. The results indicate that systems using DFH can support substantially more users than systems using RFH.**

***Index Terms*—Cellular, code-division multiple access (CDMA), dynamic channel allocation (DCA), dynamic channel assignment, EDGE, frequency hopping, orthogonal frequency-division multiplexing (OFDM), spectral efficiency, wireless.**

I. INTRODUCTION

A NUMBER of techniques have been used to combat impairments in rapidly varying radio channels and to obtain high spectral efficiencies in cellular systems. Some of those are channel coding and interleaving, adaptive modulation, transmitter/receiver antenna diversity, spectrum spreading, and dynamic channel allocation (DCA). This work utilizes a multiplicity of techniques, with an innovation in the frequency-hopping domain [1]. This paper presents the results of analytical and simulation studies of the combined effects of frequency-hopping and a special form of dynamic channel allocation (DCA) manifested by frequency-hop pattern adaptation [2]–[6].

Frequency hopping (FH) can introduce frequency diversity and interference diversity. It can be an effective technique for combating Rayleigh fading, reducing interleaving depth and associated delay, and enabling efficient frequency reuse in a multiple access communication system [7], [8], [10]. The use of frequency hopping in GSM is the most important FH application in cellular. Sony and Telia have proposed orthogonal frequency-division multiplexing (OFDM) systems with baseband types of random frequency hopping [11], [12]. Metricom has deployed a frequency-hopping-based system called Richochet. Motorola has investigated innovative applications of frequency hopping for cellular systems [13]. Ericsson has reported analytical studies and simulations of frequency-hopping GSM [14]–[16].

DCA, similarly to FH, can provide higher spectrum efficiency than fixed frequency assignment strategies, for light and medium loading.

FH and DCA techniques operate in different dimensions and thereby provide the motivation for combining them. This paper focuses on the concept, named dynamic frequency hopping (DFH), of using slow frequency hopping and adaptively modifying the utilized frequency-hop patterns based on rapid frequency quality measurements. The continuous modification of patterns based on measurements represents an application of DCA to slow frequency hopping. It is a nontraditional DCA scheme, since only a part of the channel is replaced, namely, some subset of frequencies in the frequency-hop pattern is replaced by a better quality subset. Dynamic frequency hopping attempts to track the variability of interference conditions and mitigate it by FH pattern adaptation.

An ideal DFH method works in the following way. At each hopping instant, instead of hopping randomly or according to some predetermined repetitive pattern, the base station or mobile measures the quality of each frequency, filters the measurement, and thereafter sends the data using the "best" frequencies chosen according to some criterion. Hysteresis can be applied in discarding previously used or in using new frequencies. The number of hopping frequencies that change at every hop can be limited to minimize system instabilities and complexity.

This paper contains the results of analytical and simulation studies. In the analysis part, we analyze the performance of the proposed scheme in the uplink of cellular wireless systems. The performance is expressed by outage probabilities at any frequency that is a component of a frequency-hop pattern. Motivated by large improvements in outage probabilities observed in the analytical results, we have pursued the simulation work. On the simulation front, this paper investigates both ideal and lower complexity DFH techniques:

1) full-replacement dynamic frequency hopping, which assumes changing all current frequencies of poor quality after each physical layer frame;
2) worst dwell DFH, where only one frequency (the lowest quality one) in a frequency-hop pattern is changed;
3) threshold-based DFH, where a subset of currently used frequencies is changed.

The simulations have been done for the uplink of circuit-switched cellular systems with an assumption that signal strength can be measured perfectly and that no latency is generated for the purpose of signaling. The performance is expressed by cu-

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mulative distribution functions (cdfs) illustrating the percentile of users that experience codeword error rates below some value. The results indicate that dynamic frequency hopping can improve the performance of cellular systems with frequency reuse significantly when compared to random frequency hopping.

This paper is written to provide motivation to seek solutions for increasing spectral efficiency in the domain of frequency-hop pattern adaptation. The analytical results are valid for idealized DFH, which is not implementable in a realistic cellular system. In subsequent work, we have proposed solutions that yielded implementable forms of dynamic frequency hopping requiring moderate rate measurements and less than 5% increase in signaling overhead over the air and over the inter-base- station network. The DFH methodology is particularly attractive for OFDM modulations.

Section II, together with the Appendix, presents the theoretical analysis and its results in the form of outage probabilities for random hopping and for DFH. Section III describes the simulation studies, and Section IV presents the simulation results. Section V is the conclusion.

II. Analysis of DFH

We have first pursued the theoretical analysis of DFH in order to substantiate an intuitive thinking that FH pattern adaptivity will improve the performance of frequency-hop cellular systems. Some previous studies provide excellent analyses of frequency-hopping cellular systems. We extend generic methodologies from [8] and [10] for the case of dynamic frequency hopping. Study in [5] revolves around adaptive frequency hopping using the predefined set of FH patterns with good correlation properties and is thereby more restrictive than what we are interested in. The work in [9] is custom tailored to high-frequency systems and does not easily carry into cellular systems. In this section, together with the Appendix, we derive analytical expressions suitable for comparing random frequency hopping and dynamic frequency hopping systems. We use them to verify the intuition by comparing the outage probabilities in random FH and dynamic FH cellular systems.

Slow-frequency-hopped cellular systems considered in the analysis part of this paper assume frequency reuse of one. The total available system bandwidth is divided into N frequencies. The time is slotted. The signal from a given transmitter is hopped from slot to slot by changing the carrier frequency. The hopping patterns for the users within the same cell (intracell interference) are orthogonal. Users in one cell have hopping patterns that appear to be random to users in any other cell (intercell interference). Therefore, there is no intracell interference, and the intercell interference follows a probabilistic law. We assume Reed–Solomon coding with n_s symbols, where the length of the codeword corresponds to the number of bits carrying data for one speech frame. The interleaving is done over the full frame. Hopping is done once per two code symbols.

A. Interference Modeling

We have obtained the following analytical expressions:

1) probability distribution function (pdf) of the long-term (shadowing) interference at the desired base station from one interfering user in another cell;

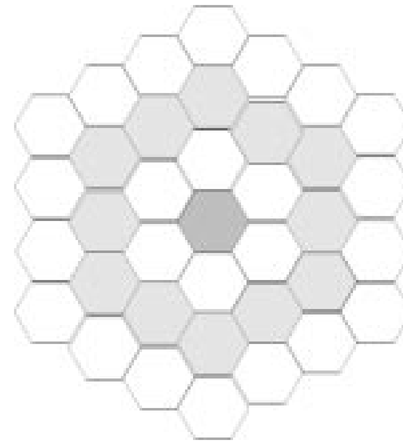

Fig. 1. A cellular system comprising hexagonal cells.

2) pdf of the combined shadowing and Rayleigh fading (modeled by a single log-normal pdf);
3) pdf of interference from the kth tier with N_k cells;
4) pdf of the total interference.

Consider a cellular system shown in Fig. 1. The central cell is the cell with the desired mobile. The interfering mobiles are in cells arranged in tiers around the central cell. The first tier has six cells, the second tier has 12 cells, the third tier has 18 cells, and so on. Each cell has a base station at its center. All cells have omnidirectional antennas.

In the analysis, we assume that the user is connected to the closest base station (in simulations, base stations with the highest power are used). Consider the interference generated from mobiles of a particular cell to the base station of interest. Fig. 2 shows the geometry. For simplicity, in what follows, we assume that each cell has a circular shape with a unit radius. Let r_j be the distance of the interfering mobile from the jth cell to the desired base and r be the distance of this mobile to its base. Let e^{ξ_j} be the shadowing attenuation of the interfering user from cell j. Then the long-term interference (caused by shadowing) at the desired base station from an interfering user in cell j is given by

$$S_j = \left(\frac{r_j}{r}\right)^{-\beta} e^{\xi_j}. \quad (1)$$

The propagation loss exponent β varies in the range 3–6. The random variable ξ_j, which models the effect of log-normal shadowing, is Gaussian distributed with zero mean and variance σ_j^2, where the value for σ varies from 6 to 12 dB.

Computation of the pdf for interference from a single mobile, caused by shadowing when power control is assumed, is shown in the Appendix. The computation assumes that the power control error can be modeled by a log-normal random variable (RV), as well as random variable S_j. The primary motivation for using log-normal distribution is to obtain tractable analytical expressions. This has been commonly done in the theoretical work in the wireless literature [22]. One should note that a better indicator of the actual system performance is obtainable only by comprehensive system simulations, presented later.

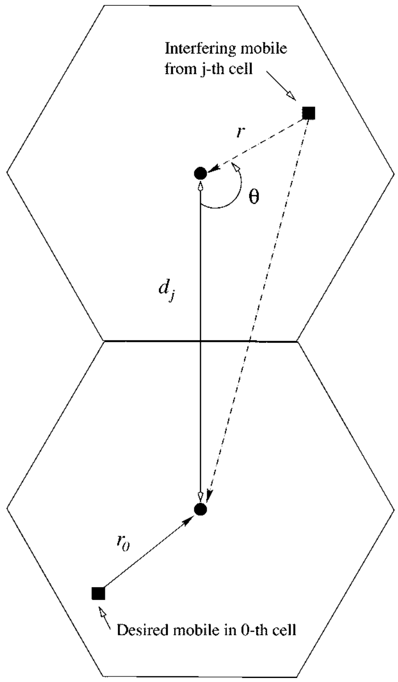

Fig. 2. Layout of the desired and interfering mobile in two cells.

The computation of the total interference arriving at the desired mobile presented in the Appendix takes into account the activity of interfering users and combines the effects of Rayleigh fading and shadowing into a single log-normal distribution [22], [24]. The computation is broken into segments, according to the tiers of surrounding cells. To compute the sum of interferences from all tiers, a lengthy convolutional expression is approximated and the final result figures as the sum of two log-normal distributions. Interference expressions for one frequency, computed until this moment, are the same for any form of frequency hopping that we further consider.

B. Outage Probability Calculations

The outage probability is defined as the probability that the signal-to-interference ratio (SIR) is less than a specified threshold, i.e., $P_{\mathrm{o}} \triangleq P[S/I < \eta]$. For the purposes of this paper, the outage probability is defined for a single hop in a set of hops, which are executed for the duration of one speech frame. Since we use Reed–Solomon coding that covers the full frame (containing several hops), we can tolerate some number of frequencies in outage condition within a frame.

1) Random FH: In the Appendix, we compute the outage probability for random hopping and a single antenna case. There, the received signal power is modeled as $S = S_0\alpha_0^2$, where α_0^2 is exponentially distributed with parameter $1/\lambda$ and S_0 models the long term signal strength fluctuation due to propagation path loss, shadowing, and power control. We provide both exact and approximate calculations for the outage probability with the desire to use the approximate expression after verifying its validity with the exact expression. Outage probabilities for the antenna diversity case are shown in [2].

2) Measurement-Based DFH: We consider two measurement-based dynamic channel allocation schemes for frequency hopping (dynamic FH pattern adaptation). Assume that there are totally N hopping frequencies. Suppose that at any hopping instant, there are $n \le N$ active users in the system. For the random hopping scheme, n out of N frequencies are picked randomly. For the measurement-based hopping schemes, however, the n frequencies are chosen according to some quality measurement. In the ideal case, we can pick the n frequencies that have the best SIR, and we call this the *best SIR* method. Alternatively, we may pick the n frequencies with the smallest interference powers, and we call this scheme the *least interference* method. Next we compute the average outage probabilities of these two hopping schemes.

In what follows, we assume that the interference from the other cells still follows the same statistics as before, i.e., as in the random hopping scheme. This is motivated by the following. In a practical system implementing dynamic frequency hopping, it will be required to have synchronized and coordinated base stations (as proposed in [17]). In that case, it will be possible to avoid undesirable situations where independent base stations simultaneously assign the same good frequencies to new frequency-hop patterns, and possibly make the performance worse. With such coordination, the outage probabilities obtained by modeling the interference frequency hopping by the random hopping scheme will be higher than outage probabilities when interference is modeled by the dynamic frequency hopping. Using the random frequency hopping for interference model also simplifies the analysis.

a) DFH Best SIR Hopping: Denote $\psi_i \triangleq (S/I)_i$ as the SIR for the ith frequency. We have previously computed the outage probability $P[\psi_i < \eta]$. Suppose that now we choose the n frequencies with the highest SIR out of the total N available. Now let $\bar{\psi}_j$ be the jth largest one in $\psi_1, \psi_2, \ldots, \psi_N$, for $j = 1, 2, \ldots, n$. Then the average outage probability in this case is given by

$$\bar{P}_{\mathrm{o}} = \frac{1}{n}\sum_{j=1}^{n} P\left[\bar{\psi}_j < \eta\right] \tag{2}$$

where the outage probability of the jth best frequency is given by

$$P\left[\bar{\psi}_j < \eta\right] = \sum_{k=N-j+1}^{N} \binom{N}{k} P_{\mathrm{o}}^k (1 - P_{\mathrm{o}})^{N-k}. \tag{3}$$

b) DFH Least Interference Hopping: In the least interference hopping scheme, the n frequencies with the least interference power are chosen in every successive hop. Denote as $\bar{I}_j$ the jth least interference power among the N frequencies; then the distribution of $\bar{I}_j$ is given by

$$F_{\bar{I}_j} = \sum_{k=j}^{M} \binom{M}{k} F_I(x)^k (1 - F_I(x))^{N-k}. \tag{4}$$

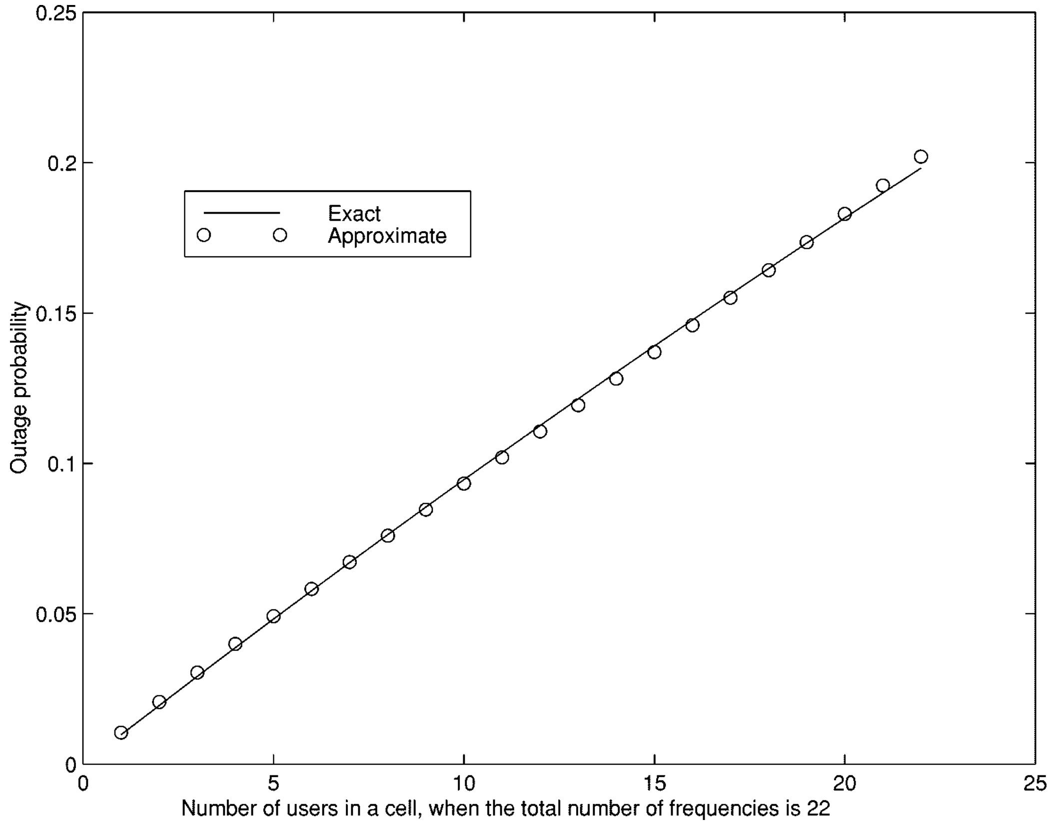

Fig. 3. Outage probability calculated for perfect power control and random hopping from the exact solution and the approximate solution.

The outage probability at the jth frequency is then given by

$$P_o(j) = P\left[\frac{S_0}{\bar{I}_j} \le \eta\right] = 1 - P\left[\bar{I}_j \le \frac{S_0}{\eta}\right] = \int_0^\infty \mathrm{d}x F_{\bar{I}_j}\left(\frac{x}{\eta}\right) f_{S_0}(x). \tag{5}$$

The average outage probability among the n active users is then given by

$$\bar{P}_o = \frac{1}{n}\sum_{j=1}^{n} P_o(j). \tag{6}$$

C. Numerical Results of the Analysis

Numerical evaluation of the analytical expressions has been obtained for the following conditions:

1) circuit-switched cellular systems with frequency reuse of one;
2) full orthogonality between frequency-hopping patterns associated with a single base station;
3) total number of available resources equal to 22 (this corresponds to 22 frequencies);
4) loading refers to the number of users that get allocated to a single cell, and can be expressed as a percentage if divided by the total number of resources;
5) three tiers of cells are considered for interference generation for all expressions using the approximation;
6) computations are done assuming circular cell shapes;
7) omnidirectional antennas are assumed;
8) outage threshold has been set at $\mathrm{SNR} = 4$ dB.

Before we use derivations from this paper, we check the validity of the approximate distribution of the interference. To do so, we compute the exact probability in a two-tier system, which employs random hopping and perfect power control, through both the exact calculation (20) and the approximate calculation (21). The outage probability is computed against the number of users in a cell, where the number of frequencies available is 22. The results are shown in Fig. 3. The match between approximate and exact results is good. In what follows, we use the approximate distribution to compute the system performance under various conditions.

Remember that for the purposes of this paper, the outage probability is defined for a single hop (frequency) in a set of hops that are executed for the duration of one speech frame. Since we use Reed–Solomon coding that covers the full frame, we can tolerate some number of frequencies in outage condition within a frame. The word error rate (WER) that is derived for one frame and on the basis of outage is a better indicator of the performance of this system. Thereby, later on, we shall use WER as a performance measure in simulations.

Fig. 4 illustrates the advantage of dynamic frequency-hopping schemes when compared to the random frequency-hopping scheme. Under ideal conditions where measurement and adaptation are assumed to be done instantaneously, the advantages are

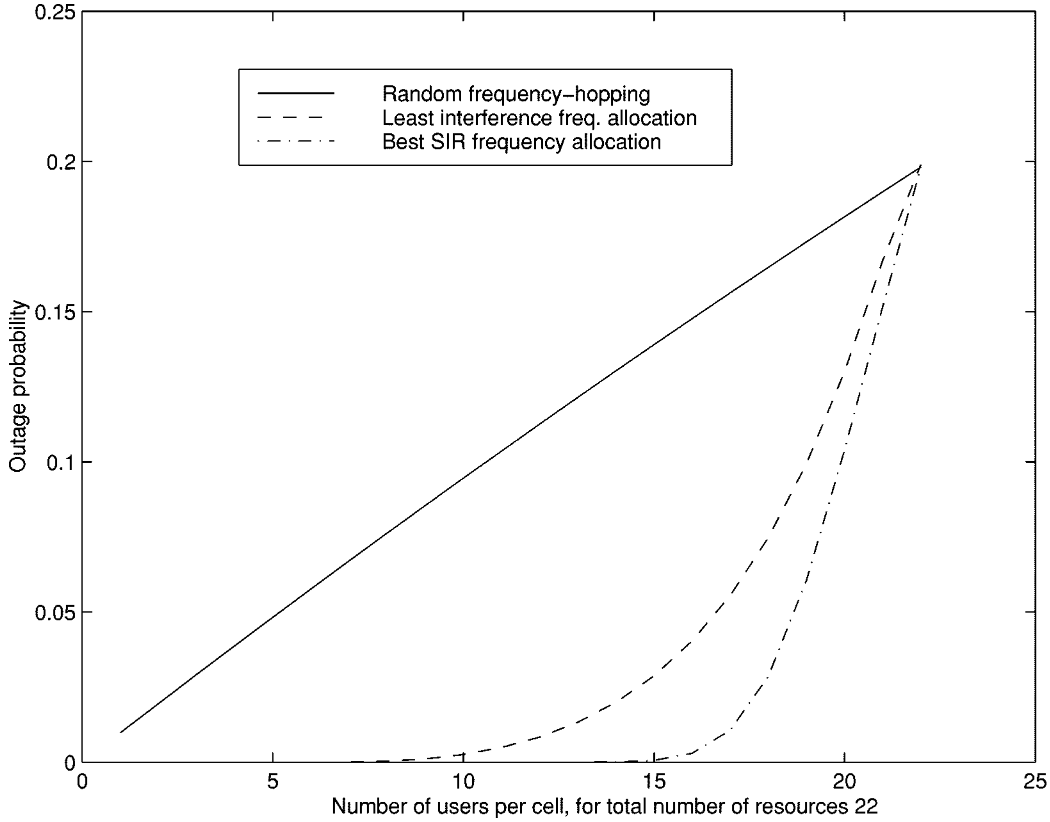

Fig. 4. Outage probability comparison for random frequency pattern allocation and for measurement-based pattern allocation.

dramatic. The least interference scheme yields slightly worse results than the best SIR frequency allocation.

Fig. 4 thereby substantiates the intuitive thought that dynamic frequency hopping outperforms random frequency hopping. Due to the approximative nature of analytical studies as well as their complexity, we pursue further evaluation of DFH by means of simulation.

III. SIMULATIONS

With the motivation provided by the analytical results, we pursued simulation studies to more accurately evaluate the benefits of dynamic frequency hopping in cellular systems. The simulations in this paper are still targeted at evaluating the potential theoretical gains of dynamic frequency hopping: we assume that measurements are ideal and encompass the effects or both shadowing and Rayleigh fading, as well as the ability to estimate the interference power. For practical systems, this is not accomplishable. We evaluated the sensitivity of the DFH performance as a function of practical constraints, came up with a practical implementation of the DFH, and presented parts of this work in [17] and [18].

This section investigates three DFH techniques:

1) full-replacement dynamic frequency hopping, which assumes changing all current frequencies of poor quality after each physical layer frame (code word);
2) worst dwell DFH, where only one frequency (the lowest quality one) in a frequency hop pattern is changed;
3) threshold-based DFH, where a subset of currently used frequencies is changed (those frequencies with SIR below a threshold).

The simulations have been done for the uplink of circuit-switched cellular systems. The performance is expressed by cdfs illustrating the percentile of users that experience code-word error rates below some value. The results indicate that dynamic frequency hopping can improve the performance of cellular systems with frequency reuse significantly, compared to random FH.

A. Description of the Simulation Model

The system evaluated in our simulations is based partly on the work by Ericsson [16]. This work is distinguished from Ericsson's in that we introduce the concept of dynamic frequency hopping, and we follow the simulation approach to obtain the results. Dynamic frequency hopping relies on continuous quality monitoring of all frequencies available in a system and modification of hopping patterns for each individual link. In practice, all frequencies can be measured if OFDM is used, or even in traditional time-division multiple access (TDMA) systems at lower speeds. In a separate paper, we address the issues of practical deployment of DFH and suggest a network-assisted methodology for reducing the overhead [17].

We consider frequency reuse of one. The total available system bandwidth of 12.8 MHz is divided into 64 frequencies (as in the GSM standard). The time is slotted into eight slots per frame. The length of one frame is 40 ms. For every frame of data from a user, a codeword that consists of 12 Reed–Solomon symbols is sent. Each pair of Reed–Solomon symbols is transmitted on a different frequency. Thus, in one frame, one user hops over six of the 64 possible frequencies. A decoder can correct no more than two pairs of symbols in an outage. If more than two pairs of symbols are in error, a frame error is declared. Hopping patterns for users within the same cell are orthogonal at all times (there is no intracell interference). Base stations in different cells are frame synchronized, but no other coordination is assumed. Thus, users in one cell have hopping patterns that appear to be random to users in any other cell (intercell interference).

For uplink frequency-hop pattern management, patterns are modified based on frequency quality measurements performed at base stations. Frequencies used in one frame are replaced by better ones in the next frame. Measurements and pattern management are done independently by each base station. There exists a problem whereby two or more base stations could identify a particular frequency as a good-quality frequency at the same time and start using it at the same moment in future. This situation would result in a high mutual interference and would be detrimental to the performance. We assume that a concept similar to token passing is deployable, which can ensure that base stations close to each other do not execute frequency modifications at the same instant of time.

Three measurement-based frequency-hopping methods were simulated.

1) *Full-Replacement Method:* All six frequencies used in one frame are replaced with better frequencies in the next frame. This guarantees that during an entire transmission, frequencies with the best quality are used. FH pattern modifications are done in a centralized fashion at each base station for all of the base station's mobiles. This method gives the best possible performance of all dynamic frequency-hopping methods. Rapid measurements of interference, SIR, or other quality variables are required for all 64 available frequencies. This method creates heavy messaging overhead over the air for exchanging the data about new frequency-hop patterns.
2) *Worst Dwell Method:* To achieve a satisfactory system performance, it is possibly enough to periodically change only one (or an arbitrary number) of the six used frequencies—the one with the worst quality (highest interference, lowest SIR, etc.).
3) *SIR Threshold-Based Method:* With this method, the pattern change is done sparingly. In each frame, SIR is measured on the six used frequencies and the current hopping pattern is changed if the measured SIR does not achieve the required threshold on at least one of them. Only the frequencies in poor conditions are changed. Any frequency that meets the threshold can be used as a replacement, and there is thus no need to scan all possible frequencies.

A hexagonal cellular system with 19 base stations has been implemented to evaluate the performance with two tiers of interferers. A cellular uplink has been modeled. A wrap-around grid structure has been devised to eliminate edge effects. The model supports loading of each cell with up to 64 uniformly distributed mobiles. Propagation attenuation is modeled using the exponential factor of 3.7 with the median path gain of -129 dB at 1 km. The model has been built with log-normal shadowing and Rayleigh fading generated with no correlation in time, but rather by drawing uncorrelated samples from the pdfs. Simulations have been done for log-normal deviation of 8 dB. Loading for all base stations is the same on average. Voice activity of 0.5 is modeled. The parameters of system simulations are summarized in Table I.

TABLE I
SIMULATION PARAMETERS

PARAMETER	VALUE
Cellular Grid	19 cells, Wrap around
Cells	Hexagonal, Unit radius
Simulation Type	Static, Uplink
Interference	Random, Two tiers
Propagation	exp = 3.7, path gain 129dB at 1Km
Shadowing Variance	8dB
Freq. Quality Threshold	4dB
Power Control	Path gain (shadowing) compensation
Frequencies Available	64
Channel Coding	Reed Solomon - 12 smbls/frame, 2 smbls/dwell
Receiver	2 branch max ratio combining
Occupancy	$0-64$ mobiles per cell
Voice Activity	0.5
Mobile Distribution	Uniform
Mobile Assignment	Base with highest power

For dynamic frequency-hopping simulations, FH patterns are reallocated independently by each base station, making sure that FH patterns of all mobiles within one base are orthogonal.

A result of one simulation is a cumulative distribution function of the link word error rate, obtained across all base stations for all active mobiles. Thermal noise is ignored. Actual Reed–Solomon decoding is not executed in the simulations. Rather, a simple count of the number of symbols in outage (per frame) is used to evaluate if the decoder could recover the frame from errors. If more than two frequencies (out of six total) used in a frequency-hop pattern (one per frame) are in poor conditions, it is declared that the word is in error. We use the threshold-based decision where the SIR threshold is set to 4 dB, a number appropriate when Rayleigh fading is included in simulations. The resulting cdf is truncated, since all WERs smaller than 0.001 are accumulated in one point of the cdf function.

Two Rayleigh fading models have been used in the experiments. In one model, each of the frequencies in a frequency-hop pattern is faded independently. This model is reasonable for systems where the total bandwidth available for hopping is large enough to guarantee frequency decorrelation. Such would be the case for the GSM system if 12.8 MHz were used. Another model assumes that for one user, all frequencies in a frame experience the same Rayleigh fading. This model takes advantage only of the interference avoidance property of frequency-hopping, and it is suitable for systems where the total available bandwidth

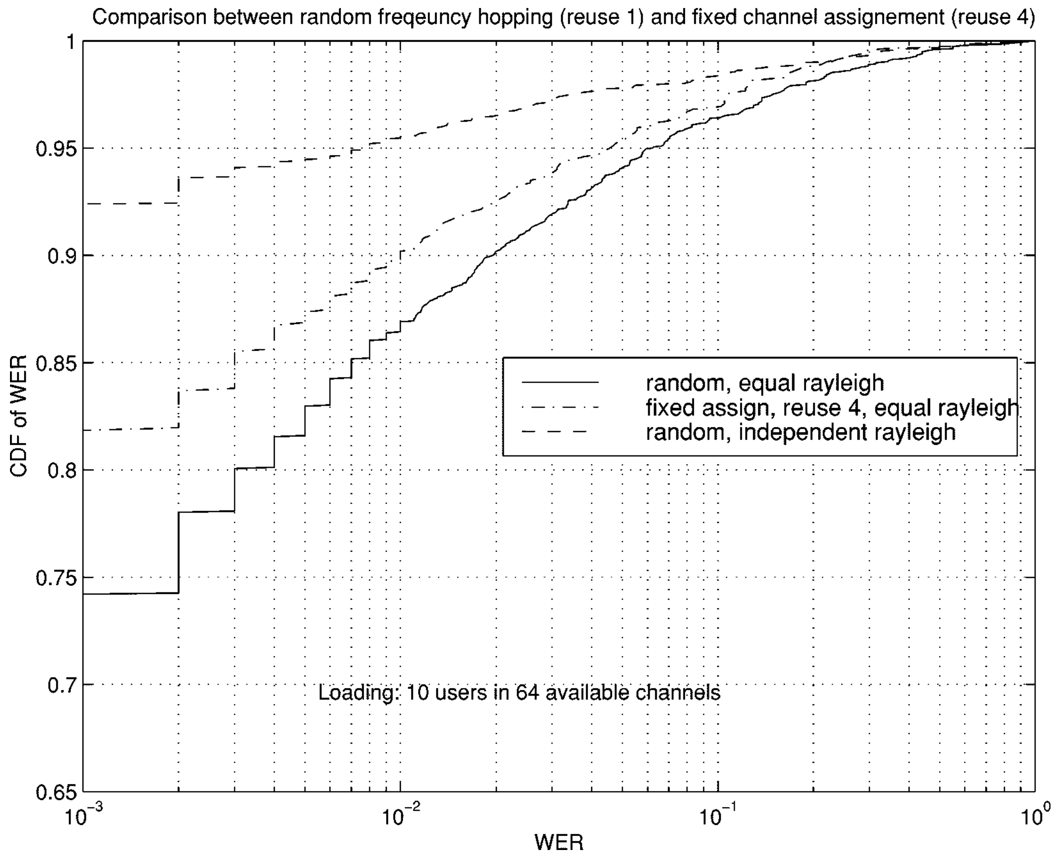

Fig. 5. Fixed channel allocation versus random frequency hopping.

is on the order of channel coherence bandwidth. In practical wide-area cellular systems, the coherence bandwidth is on the order of 1–2 MHz and the total bandwidth for hopping may be between 1 and 10 MHz. Both models have been evaluated and compared. From the simulation time perspective, the model where all frequencies fade the same is less complex. Since the primary target of this work is to establish the benefits of interference avoidance by dynamic frequency hopping, most of the simulations are done with common (fully correlated) Rayleigh fading.

nch receive antenna diversity has been incorporated in the simulations by default. Both omnidirectional antenna and three-sector sectorized cell cases have been evaluated. A comparison has been done between the two, but the majority of the results shown in this paper are for the omnidirectional case. For purposes of comparison, a cellular system with fixed channel assignment and frequency reuse of four has been implemented. Simulations have been run for single slot operation, which suggests that a crude way of calculating the effective bits/second/hertz performance can be done by multiplication with a factor of eight (eight slots per frame).

IV. Simulation Results

The validity of the proposed simulation model has been verified by comparing random frequency hopping with frequency reuse of one and fixed channel assignment with frequency reuse of four. Comparisons have been made for GSM-like environments and for two values of occupancy—eight and ten users per cell per slot per full band (8/64 and 10/64, where 64 indicates the total number of available frequencies). Results for 10/64 are shown in Fig. 5.

The figure should be read as follows. 1) Every curve represents the performance for one set of parameters. 2) A curve is a cumulative distribution function for the set of word error rates attained by all users in the system for a simulation run. 3) One point on the uppermost curve communicates that a given percentage (ratio below one) of users has the word error rate lower than that point's x-axis value. 4) The higher up the curve is on the graph, the better the performance for corresponding parameters is.

As expected, in Fig. 5, for equally faded frequencies, the random hopping model performs slightly worse than the fixed channel assignment. For independently fading frequencies, random hopping is superior. At the occupancy of 10/64, the fixed channel assignment can achieve 90 percentile coverage with word error rates better than $10e^{-2}$. To get the approximate value of spectral efficiency, we multiply the number of users per cell per slot per total bandwidth with the number of slots and divide this by the total bandwidth: 10 [users/cell/slot]^{*8}[slots]$/12.8$ [MHz] $=$ 6.25 [users/cell/MHz]. For the same spectral efficiency, the coverage of random frequency hopping is 95 percentile, for independent Rayleigh fading on all frequencies. These results are reasonable when compared to the actual GSM numbers, remembering that the results here are shown for omnidirectional antennas and that channel coding is different from what GSM systems use. Comparing our random hopping results with Two-

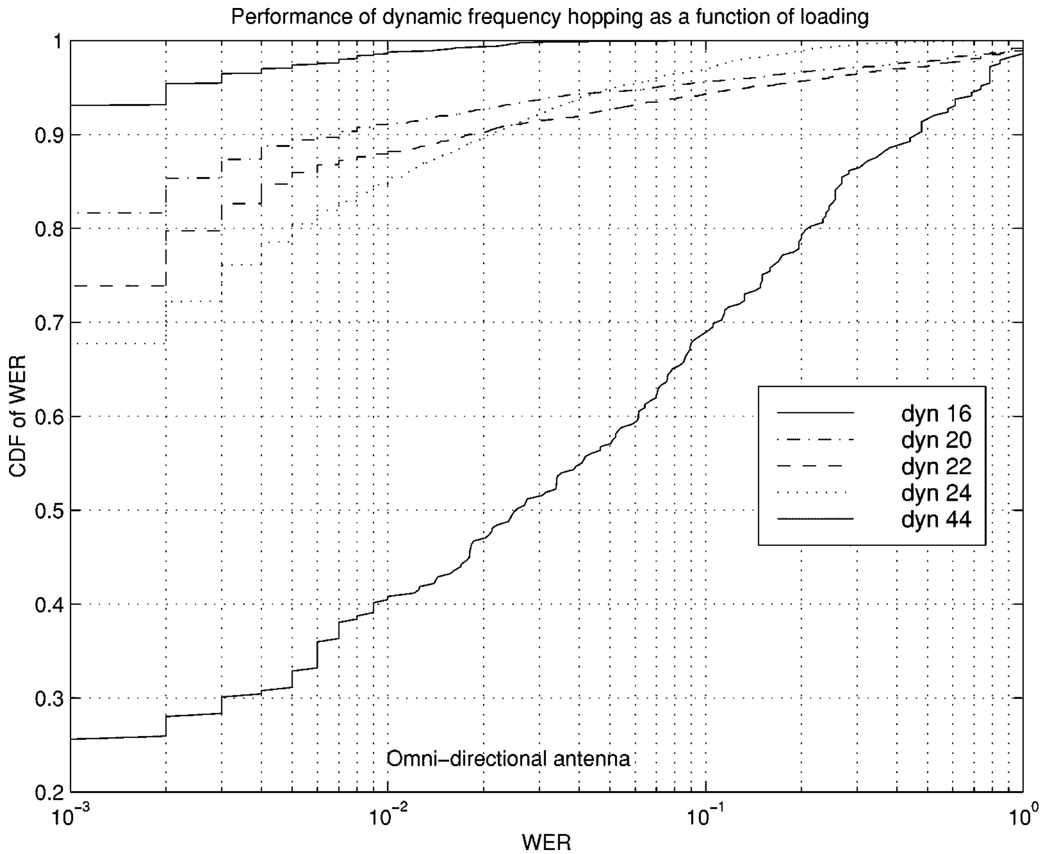

Fig. 6. Performance of full-replacement dynamic frequency hopping as a function of occupancy.

brathe results from Ericsson's paper [16], we note that in Fig. 1, Chennakeshu reports around 17 [users/cell/MHz] for similar conditions. Simulations presented in the figures above do not utilize power control and antenna sectorization, both of which bring significant improvements (shown later).

A. Performance of the Full Replacement FH Method

Fig. 6 shows the performance of the full replacement dynamic frequency hopping for several occupancies between 16 mobiles per cell per slot per total bandwidth and 44 mobiles per cell per slot per total bandwidth (16/64 and 44/64). The results apply to the same Rayleigh fading in all hops. Both 16/64 and 20/64 occupancies provide better than $10e^{-2}$ WER at 90 percentile coverage. This maps into the spectral efficiency of $20*8/12.8 = 12.5$ [users/cell/MHz].

In Fig. 7, we contrast the performance of random frequency hopping and dynamic frequency hopping for occupancies of 16/64 and 24/64 with identical Rayleigh fading for all hops.

The benefits of independent Rayleigh fading are illustrated in Fig. 8 for occupancy of 24/64. For DFH, occupancy of 24/64 will result in 98 percentile coverage at WER lower than $10e^{-2}$.

To illustrate the benefits of three-sector antenna sectorization, we plot Fig. 9. Observe that even for random frequency hopping, the WER of $10e^{-2}$ can be reached for 96% of users for occupancy of 16/64 and for three-sector antenna sectorization. The results are comparable to those in [16].

B. Performance of Low Complexity DFH Methods

Fig. 10 gives the comparison of three dynamic FH pattern adaptation methods and random hopping for occupancy of 16/64 and identical Rayleigh fading in all six hops. We observe the improvement in performance compared to random hopping. The SIR threshold for the threshold-based method is set to 8 dB.

C. Comments

Implementation of ideal dynamic frequency hopping, which requires intensive measurement processes and significant overhead for signaling, is impossible in realistic cellular systems. To address this issue, we have investigated methodologies for deploying DFH in realistic systems, as well as the impact of reduced rate measurements and signaling latencies. This approach requires base-station synchronization and coordination [17]. The results of this paper provided motivation to seek implementable solutions for dynamic frequency hopping.

Some additional issues that we investigated in the course of the DFH study are as follows.

1) Reference [3] contains a large set of curves illustrating various aspects of the performance of low-complexity DFH methods.
2) Setting SNR threshold for detection to a value other than 4 dB does not change the relative comparison between RFH and different forms of DFH.

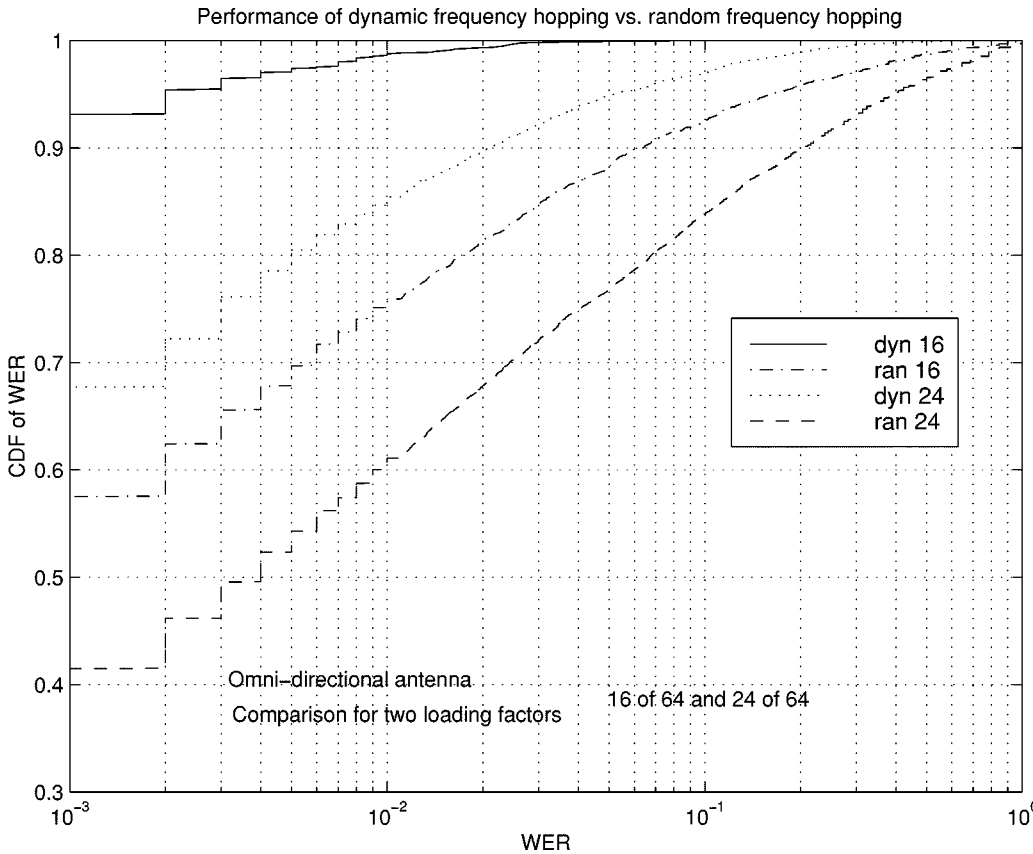

Fig. 7. Comparative performance of full-replacement dynamic and random frequency hopping.

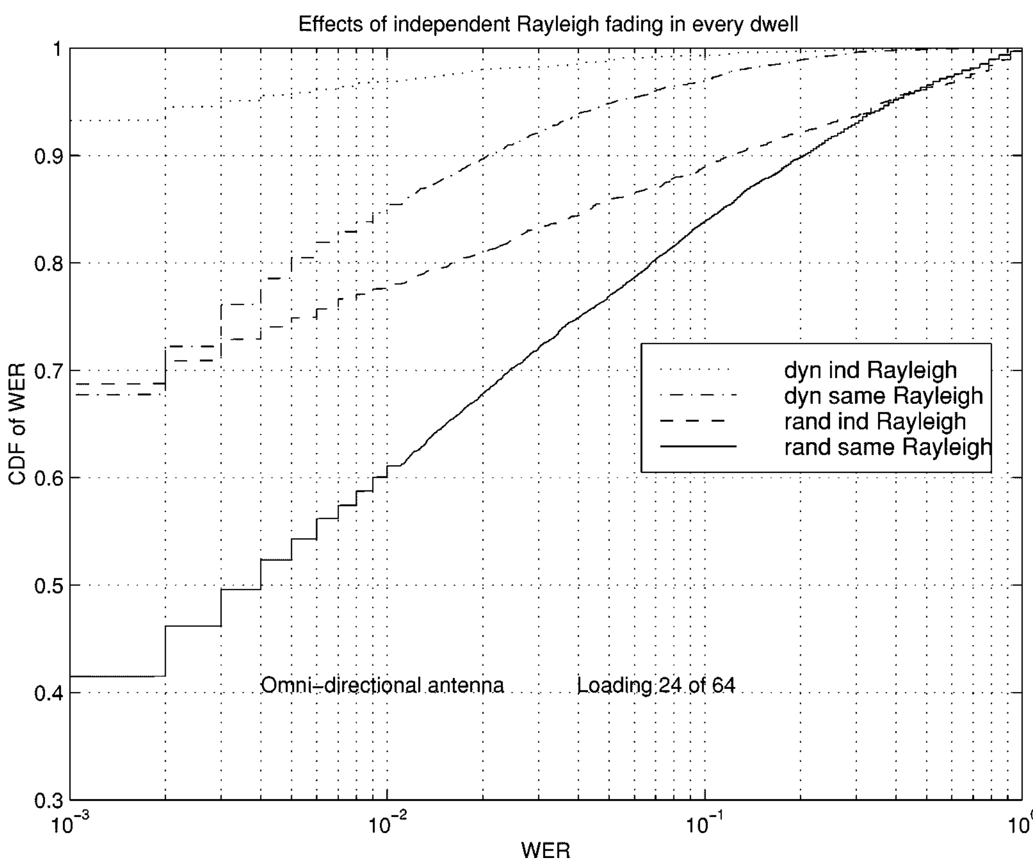

Fig. 8. Comparison of effects of independent and common Rayleigh fading for all frequencies in a slot.

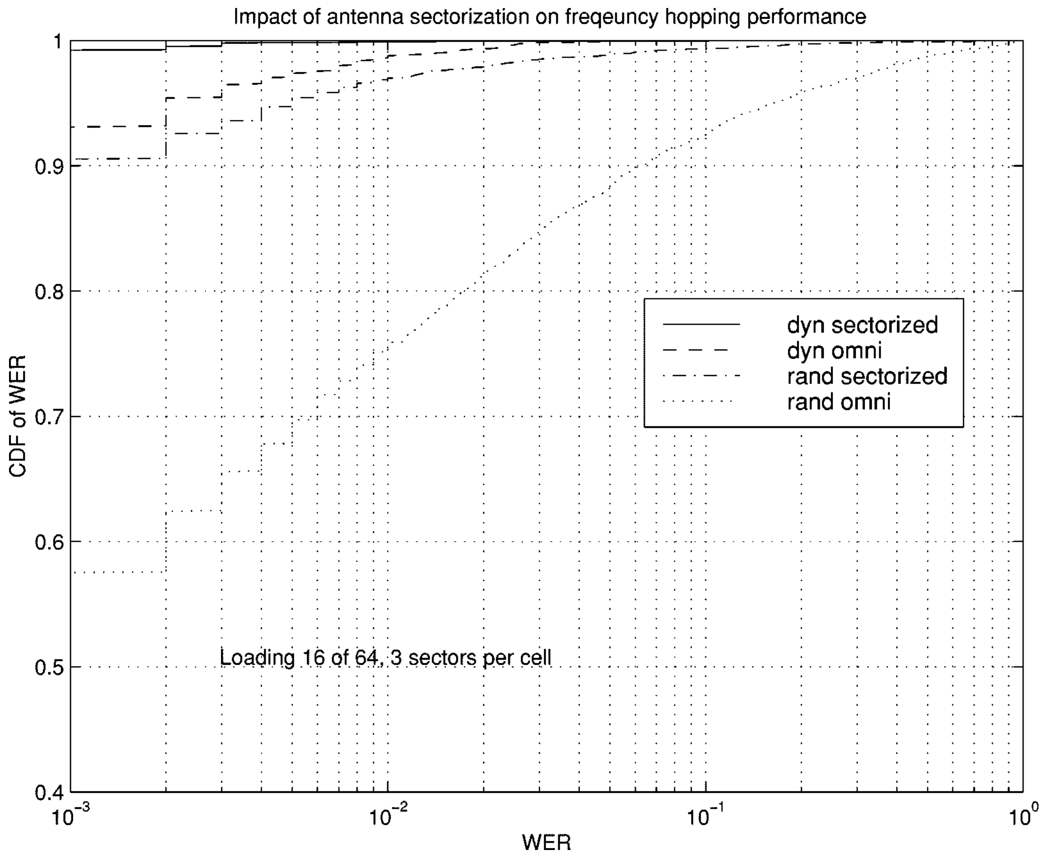

Fig. 9. Performance in systems with and without antenna sectorization for full replacement dynamic and random frequency hopping.

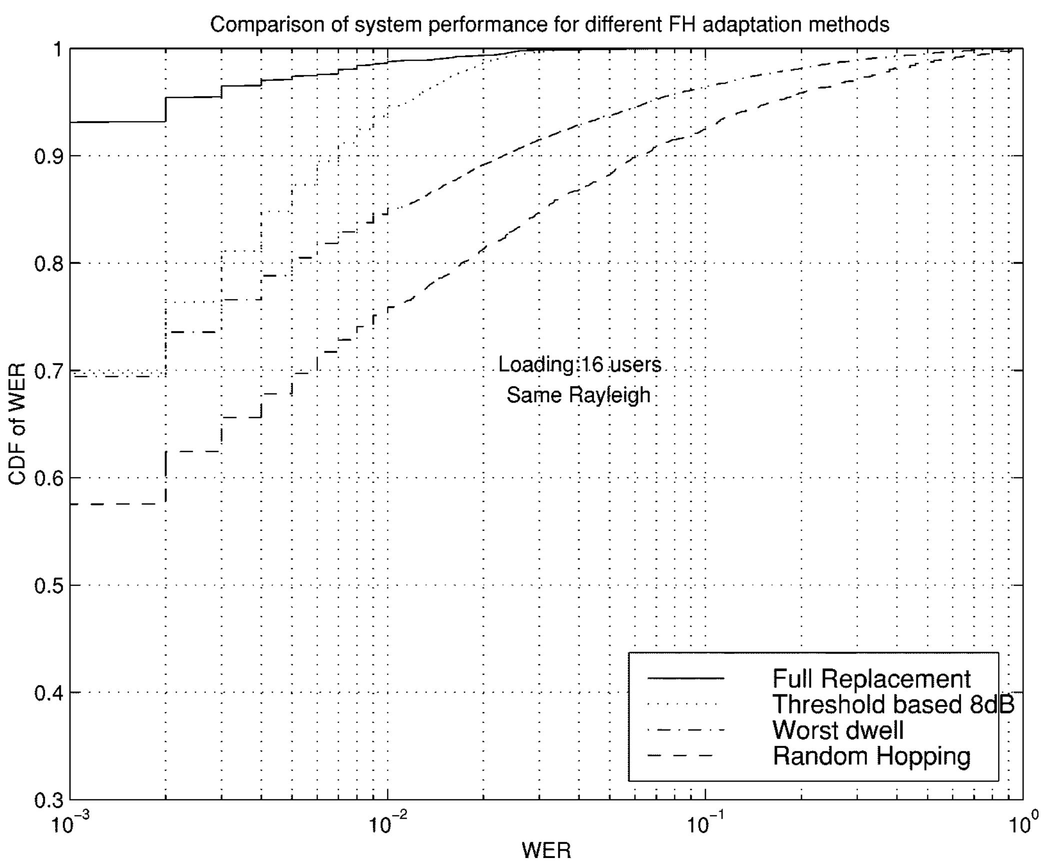

Fig. 10. Comparison of system performance for different FH adaptation methods.

V. Conclusion

This paper evaluates the performance of the measurement-based frequency-hop pattern adaptation (dynamic frequency hopping) applied to cellular systems.

We have developed an analytical framework for modeling cellular systems with frequency reuse of one that takes into account mobile locations, propagation path loss, shadowing, multipath fading, Doppler shift, channel correlation, antenna diversity, channel coding, and dynamic channel allocation. Through the numerical evaluation of the outage probabilities per one frequency-hop dwell, we have obtained the results that indicate that DFH, in ideal conditions, significantly outperforms random frequency hopping. For a loading factor of 0.8, the DFH creates an outage of 5%, whereas random frequency hopping generates an outage of 15%.

We have also generated simulation results that illustrate the gains of DFH presented through cumulative distribution functions of the word error rate. Three dynamic frequency-hopping techniques have been investigated: 1) full-replacement dynamic frequency hopping, b) worst dwell dynamic frequency hopping, and c) threshold-based dynamic frequency hopping.

As an example, the results indicate that the full-replacement dynamic frequency hopping provides service to 90% of users at WER better than $10e^{-2}$ with occupancy of 21/64 (21 users per cell per slot per total bandwidth). This corresponds to the spectral efficiency of 13.1 [users/cell/MHz] and can also be presented as the occupancy of 32.8%. The same coverage and WER are obtained by random frequency hopping for the occupancy below 12.5%. In both cases, power control and antenna sectorization are not considered. The performance of the reduced-overhead methods is worse than the performance of the full-replacement method but better than the performance of random frequency hopping. The choice of thresholds and the number of frequencies that get changed in one adaptation step can provide systems with performance anywhere in the range between full-replacement dynamic frequency hopping and random frequency hopping. We also show that the improvements in performance are significant when independent Rayleigh fading is assumed for frequencies in a frequency-hop pattern.

Appendix

A. PDFs for Interference From a Single Mobile

Assume that the interfering mobiles are uniformly distributed in their cell j. Let d_j be the distance between the base station j and the desired base station, and let (r, θ) be the polar coordinate of the interferer in cell j. We consider the distribution of the RV S_j when there is power control. For the desired signal, the effect of power control is to normalize the mean signal strength to unity and reduce the shadowing variance to the power control error. For each interfering signal, the effect of power control is to scale the area mean signal strength and increase the power variation by adding a power control error to the shadowing factor. A method for modeling power control error, which is used in the literature [16], is to represent it by a log-normal RV, e^{ξ_e}, where $\xi_e \sim \mathcal{N}(0, \sigma_e^2)$. Therefore, the distribution of S_j under power control is given by

$$\begin{aligned}
&f_{S_j}(x)\\
&= \frac{\partial}{\partial x} P\left[S_j \le x\right]\\
&= \frac{1}{\pi}\int_0^{2\pi} \mathrm{d}\theta \int_0^1 \mathrm{d}r r \frac{\partial}{\partial x}\\
&\quad \times P\left[\left(r^2 + d_j^2 - 2rd_j\cos\theta\right)^{-\beta/2} r^\beta e^{\xi_0+\xi_e} \le x\right]\\
&= \frac{1}{\pi}\int_0^{2\pi} \mathrm{d}\theta \int_0^1 \mathrm{d}r\, r \frac{\partial}{\partial x}\Phi\\
&\quad \times \left(\frac{\log(x) + \frac{\beta}{2}\log\left(1 + \left(\frac{d_j}{r}\right)^2 - 2\left(\frac{d_j}{r}\right)\cos\theta\right)}{\sqrt{\sigma^2+\sigma_e^2}}\right)\\
&= \frac{1}{\sqrt{2\pi^3\left(\sigma^2+\sigma_e^2\right)}x}\int_0^{2\pi} \mathrm{d}\theta \int_0^1 \mathrm{d}r\, r \exp\\
&\quad \times \left(-\frac{\left[\log(x) + \frac{\beta}{2}\log\left(1 + \left(\frac{d_j}{r}\right)^2 - 2\left(\frac{d_j}{r}\right)\cos\theta\right)\right]^2}{2\left(\sigma^2+\sigma_e^2\right)}\right)
\end{aligned}$$

where $\Phi(\cdot)$ is the cumulative Gaussian distribution function, i.e., $\Phi(x) = 1/\sqrt{2\pi}\int_{-\infty}^{x} e^{-t^2/2} \cdot \mathrm{d}t$.

As the first step toward approximating the distribution of the total interference, we approximate the distribution of S_j by a log-normal distribution, i.e., $S_j \cong e^{X_j}$, $X_j \sim \mathcal{N}\left(m_j, u_j^2\right)$, where m_j and u_j^2 are the logarithmic area mean power and logarithmic variance of the log-normal approximate distribution

$$m_j \triangleq E\left\{\log(S_j)\right\} = \int_{-\infty}^{\infty} x e^x f_{S_j}(e^x)\mathrm{d}x \tag{7}$$

$$u_j^2 \triangleq E\left\{\log^2(S_j)\right\} - m_j^2 = \int_{-\infty}^{\infty} x^2 e^x f_{S_j}(e^x)\mathrm{d}x - m_j^2. \tag{8}$$

We modeled both power control error and the interference by log-normal variables.

B. PDFs for Total Interference

The total interference arriving at the desired mobile can be written as

$$I = \sum_{j\in\mathcal{T}_1} \chi_j S_j \alpha_j^2 + \sum_{j\in\mathcal{T}_2} \chi_j S_j \alpha_j^2 + \cdots \tag{9}$$

where

- $\mathcal{T}_j$ — jth tier;
- χ_j — Bernoulli RV representing the activity of a user at the considered frequency (voice activity in circuit switched systems) in the jth cell;
- α_j — exponentially distributed with parameter $1/\lambda$.

The effect of combined log-normal shadowing and Rayleigh fading is approximated by a single log-normal distribution [22], i.e., $Z_j \triangleq S_j \alpha_j^2 \cong e^{Y_j}, Y_j \sim \mathcal{N}\left(\mu_j, \upsilon_j^2\right)$. Furthermore

$$\begin{aligned} F_{Z_j}(z) =& 1 - P(Z_j > z) \\ =& 1 - \int_0^\infty P\left(S_j \alpha_j^2 > z \mid S_j = s\right) f_{S_j}(s) \mathrm{d}s \\ =& 1 - \int_0^\infty \exp\left(-\frac{z}{\lambda s}\right) f_{S_j}(s) \mathrm{d}s \qquad (10) \\ f_{Z_j}(z) \triangleq& \frac{\partial F_{Z_j}(z)}{\partial z} \\ \cong& \frac{1}{\sqrt{2\pi}\lambda u_j} \int_0^\infty \frac{1}{s^2} \\ & \times \exp\left(-\frac{(\log(s) - m_j)^2}{2u_j^2} - \frac{z}{\lambda s}\right) \mathrm{d}s. \qquad (11) \end{aligned}$$

Now μ_j and υ_j^2 are given by

$$\mu_j \triangleq E\left\{\log(Z_j)\right\} = \int_{-\infty}^\infty y e^y f_{Z_j}(e^y) \mathrm{d}y \qquad (12)$$

$$\upsilon_j^2 \triangleq E\left\{\log^2(Z_j)\right\} - \mu_j^2 = \int_{-\infty}^\infty y^2 e^y f_{Z_j}(e^y) \mathrm{d}y - \mu_j^2. \qquad (13)$$

The distribution of the total interference power from the kth tier with N_k cells is

$$I_k \triangleq \sum_{j=1}^{N_k} \chi_j^k S_j^k {\alpha_i^k}^2 = \sum_{i=1}^{N_k} \chi_j^k Z_i^k.$$

Note that Z_i^k are independent and identically distributed (i.i.d.) RV. Suppose that $P(\chi_i) = p$; then it is easily seen that the distribution of I_k is given by

$$f_{I_k}(x) = (1-p)^{N_k} \delta(x) + \sum_{i=1}^{N_k} \binom{N_k}{i} (1-p)^{N_k - i} p^i f^i_{Z_1^k}(x) \qquad (14)$$

where $f^i_{Z_1^k}$ denotes the pdf of a sum of i i.i.d. RVs, each having the same log-normal distribution as Z_1^k. It is a common practice to approximate the distribution of a sum of log-normal RVs by another log-normal RV. A recursive technique for estimating the first two moments of the logarithm of a sum of $i \geq 2$ independent log-normal RVs can be found in [24]. By applying this procedure, starting with the logarithmic mean μ_{1k} and logarithmic variance υ_{2k}^2 of the distribution $f^1_{Z_1^k}$, we can then compute the logarithmic mean μ_{ik} and logarithmic variance υ_{ik}^2 of the log-normal pdf $f^i_{Z_1^k}$. Therefore the total distribution from the kth tier $f_{I_k}(x)$ is obtained. Suppose that there are a total of K tiers that contribute interference at the desired cell. Then the distribution of the total interference $I \triangleq \sum_{k=1}^K I_k$ is given by

$$f_I(x) = f_{I_1}(x) \otimes f_{I_2}(x) \otimes \cdots \otimes f_{I_K}(x) \qquad (15)$$

where $\otimes$ denotes convolution. A direct substitution of (14) into (15) would result in an extremely lengthy expression. Therefore, we need a further approximation on (15). To that end, we approximate the summation term on the right-hand side of (14) by a new log-normal distribution function, i.e.,

$$\sum_{i=1}^{N_k} \binom{N_k}{i} (1-p)^{N_k - i} p^i f_k^i(x) \cong \left[1 - (1-p)^{N_k}\right] g_k(x) \qquad (16)$$

with $g_k(x)$ a log-normal distribution function. The logarithmic mean and logarithmic variance of the new distribution g_k can be calculated as follows. Suppose that $U_k = e^{T_k} \sim g_k(x)$; then

$$\begin{aligned} E(T_k) =& \int_{-\infty}^\infty t e^t g_k(e^t) \mathrm{d}t \\ =& \frac{1}{\left[1 - (1-p)^{N_k}\right]} \sum_{i=1}^{N_k} \binom{N_k}{i} (1-p)^{N_k - i} \\ & \times p^i \int_{-\infty}^\infty t e^t f_k^i(e^t) \mathrm{d}t \\ E(T_k^2) =& \int_{-\infty}^\infty t^2 e^t g_k(e^t) \mathrm{d}t \\ =& \frac{1}{\left[1 - (1-p)^{N_k}\right]} \sum_{i=1}^{N_k} \binom{N_k}{i} (1-p)^{N_k - i} \\ & \times p^i \int_{-\infty}^\infty t^2 e^t f_k^i(e^t) \mathrm{d}t. \end{aligned}$$

Denote $q_k \triangleq (1-p)^{N_k}$; then (14) is approximated by

$$f_{I_k}(x) \cong q_k \delta(x) + (1 - q_k) g_k(x). \qquad (17)$$

Now substituting (17) into (15), we obtain the total interference distribution. For instance, the distribution of the total interference from the first two tiers $I = I_1 + I_2$ is given by

$$\begin{aligned} f_I(x) =& \left[q_1 \delta(x) + (1 - q_1) g_1(x)\right] \\ & \otimes \left[q_2 \delta(x) + (1 - q_2) g_2(x)\right] \\ =& q_1 q_2 \delta(x) + (1 - q_1) q_2 g_1(x) \\ & + q_1 (1 - q_2) g_2(x) (1 - q_1)(1 - q_2) g_1 \otimes g_2(x). \end{aligned} \qquad (18)$$

$g_1 \otimes g_2(x)$ is the distribution function of the sum of two independent log-normal RVs, and thus it is a log-normal distribution function with mean and variance computable by the method in [24].

C. Outage Probability for Random Hopping

The received signal power is modeled as $S = S_0 \alpha_0^2$, where α_0^2 is exponentially distributed with parameter $1/\lambda$ and S_0 models the long-term signal strength fluctuation due to propagation path loss, shadowing, and power control. With power control, the pdf of S_0 is given by

$$f_{S_0}(x) = \frac{\partial}{\partial x} P\left[e^{\xi_e} \leq x\right] = \frac{1}{\sqrt{2\pi}\sigma_e x} \exp\left(-\frac{\log^2(x)}{2\sigma_e^2}\right). \qquad (19)$$

An exact closed expression for the outage probability can be obtained as follows:

$$\begin{aligned} P_\mathrm{o} \triangleq& P\left[\frac{S_0 \alpha_0^2}{\sum_j \chi_j S_j \alpha_j^2} < \eta\right] \\ =& 1 - P\left[\alpha_0^2 > \eta \sum_j \chi_j \frac{S_j}{S_0} \alpha_j^2\right] \end{aligned}$$

$$
\begin{aligned}
&=1-E_{S_0,\chi_j,S_j,\alpha_j^2}\left[\exp\left(-\frac{\eta}{\lambda}\sum_j \chi_j\frac{S_j}{S_0}\alpha_j^2\right)\right]\\
&=1-E_{S_0}\left[\prod_j E_{\chi_j,S_j}\left[E_{\alpha_j^2}\left[\exp\left(-\frac{\eta}{\lambda}\chi_j\frac{S_j}{S_0}\alpha_j^2\right)\right]\right]\right]\\
&=1-E_{S_0}\left[\prod_j\left(1-p+pE_{S_j}\left[\frac{1}{1+\eta\frac{S_j}{S_0}}\right]\right)\right]. \qquad (20)
\end{aligned}
$$

If we use the approximate distribution of the total interference I developed in the previous section, we have

$$
\begin{aligned}
P_{\mathrm{o}} &\triangleq P\left[\frac{S_0\alpha_0^2}{I}<\eta\right]\\
&=1-P\left[\alpha_0^2>\eta\frac{I}{S_0}\right]\\
&\cong 1-\int_0^\infty \mathrm{d}x\int_0^\infty \mathrm{d}y\exp\left(-\frac{x}{\lambda y}\right)f_{S_0}(x)f_I(y).
\end{aligned}
\qquad (21)
$$

REFERENCES

[1] R. L. Pickholtz *et al.*, "Theory of spread spectrum communications–a tutorial," *IEEE Trans. Commun.*, vol. COM-30, pp. 855–884, May 1982.

[2] X. Wang and Z. Kostic, "Analysis of frequency-hopped cellular systems with dynamic FH pattern adaptation," in *Proc. IEEE Globecom, Communications Theory Mini Conf.*, Sydney, Australia, Nov. 1998.

[3] Z. Kostic and I. Maric, "Dynamic frequency hopping in wireless cellular systems—simulations of full-replacement and reduced-overhead methods," in *Proc. IEEE VTC'99*, Houston, TX, May 1999.

[4] Q. C. Chen *et al.*, "Multicarrier CDMA with adaptive frequency hopping for mobile radio systems," *IEEE J. Select. Areas Commun.*, vol. 14, pp. 1852–1858, Dec. 1996.

[5] C. C. Wang and G. Pottie, "Interference avoidance and power control strategies for coded frequency hopped cellular systems," in *Proc. IEEE Int. Conf. Communications (ICC)*, 1995, pp. 1737–1741.

[6] J. Deissner and G. P. Fettweis, "A study on hierarchical cellular structures with inter-layer reuse in a enhanced GSM radio network," in *Proc. Conf. Mobile Multimedia Communications (MoMuC99)*, 1999, pp. 243–251.

[7] J. L. Dornstetter and D. Verhulst, "Cellular efficiency with slow frequency hopping: Analysis of the digital SFH 900 mobile system," *IEEE J. Select. Areas Commun.*, vol. SAC-5, pp. 835–848, July 1987.

[8] C. C. Lee and R. Steele, "Signal to interference calculations for modern TDMA cellular communication systems," *Proc. Inst. Elect. Eng. Commun.*, vol. 142, pp. 21–30, Feb. 1995.

[9] J. Zander and G. Malmgren, "Adaptive frequency hopping in HF communications," *Proc. Inst. Elect. Eng. Commun.*, vol. 142, pp. 99–105, Apr. 1995.

[10] M. Chiani *et al.*, "Outage evaluation for slow frequency-hopping mobile radio systems," *IEEE Transactions on Communications*, vol. 47, pp. 1865–1874, Dec 1999.

[11] "Sony International (Europe) GmbH, BDMA, the multiple access scheme proposal for the UMTS terrestrial radio air interface (UTRA)," in *ETSI SMG2 Concept Group Meeting*, London, U.K., June 23–27,1997.

[12] M. Wahlqvist *et al.*, "A Conceptual study of OFDM-based multiple-access schemes part 1 to 5," Telia Research, Lulea, Sweden, 1996.

[13] P. Rasky *et al.*, "Slow frequency-hop TDMA/CDMA for macrocellular personal communications," *IEEE Commun. Mag.*, vol. 1, no. 2, pp. 26–35, 2nd quarter 1994.

[14] H. O. Olofsson *et al.*, "Interference diversity gain in frequency hopping GSM," in *Proc. 45th IEEE Vehicular Technology Conf.*, 1995, pp. 102–106.

[15] C. C. Carneheim *et al.*, "Frequency hopping GSM," in *Proc. 44th IEEE Vehicular Technology Conf.*, 1994, pp. 1155–1159.

[16] S. C. Chennakeshu *et al.*, "Capacity analysis of a TDMA-based slow-frequency-hopped cellular system," *IEEE Trans. Veh. Technol.*, vol. 45, pp. 531–542, Aug. 1996.

[17] Z. K. Kostic *et al.*, "Dynamic frequency hopping in cellular systems with network assisted resource allocation," in *Proc. Spring 2000 IEEE Vehicular Technology Conf.*, Tokyo, Japan, 2000.

[18] ——, "Dynamic frequency hopping for limited-bandwidth cellular systems," in *Proc. 9th Int. Conf. Computer Communications and Networks*, Las Vegas, NV, Oct. 2000, pp. 28–33.

[19] W. C. Jakes Jr, *Microwave Mobile Communications*, 3rd ed. New York, 1974.

[20] E. N. Gilbert, "Capacity of a burst-noise channel," *Bell Syst. Tech. J.*, vol. 39, pp. 1253–1266, Sept. 1960.

[21] G. J. Pottie, "System design choices in personal communications," *IEEE Commun. Mag.*, pp. 50–67, Oct 1995.

[22] R. Prasad and A. Kegel, "Improved assessment of interference limits in cellular radio performance," *IEEE Trans. Veh. Technol.*, vol. 40, pp. 412–419, Feb 1991.

[23] W. Turin, *Performance Analysis of Digital Transmission Systems*. New York: Computer Science, 1990.

[24] S. C. Schwartz and Y. S. Yeh, "On the distribution function and moments of power sums with log-normal components," *Bell Syst. Tech. J.*, vol. 61, pp. 1441–1462, Nov. 1982.

[25] D. Verhulst, M. Mouly, and J. Szpirglas, "Slow frequency hopping multiple access for digital cellular radiotelephone," *IEEE J. Select. Areas Commun.*, vol. SAC-2, pp. 563–574, July 1984.

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