Multihop Virtual Full-Duplex Relay Channels

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***Abstract*—We introduce a multihop "virtual" full-duplex relay channel as a special case of a general multiple multicast relay network. For such network, quantize-map-and-forward (QMF) (or noisy network coding (NNC)) can achieve the cut-set upper bound within a constant gap where the gap grows *linearly* with the number of relay stages K. However, this gap may not be negligible for the systems with multihop transmissions (e.g., a power-limited wireless backhaul system operating at high frequencies). In this paper, we obtain an improved result to the capacity scaling where the gap grows *logarithmically* as $\log(K)$. This is achieved by using an optimal quantization at relays and by exploiting relays' messages (decoded in the previous time slot) as side-information at the destination. We further improve the performance of this network by presenting a *mixed* strategy where each relay can perform either decode-and-forward (DF) or QMF with possibly *rate-splitting*.**

***Index Terms*—Multihop relay networks, half-duplex relays, quantize-map-and-forward**

I. Introduction

Recent works have demonstrated the practical feasibility of full-duplex relays through the suppression of self-interference in a mixed analog-digital fashion in order to avoid the problem of receiver power saturation [1]. These architectures are based on some form of analog self-interference cancellation, followed by digital self-interference cancellation in the baseband domain. In some of these architectures, the self-interference cancellation in the analog domain is obtained by transmitting with multiple antennas such that the signals transmitted over different antennas superimpose in opposite phases and therefore cancel each other at the receiving antennas. Building on the idea of using multiple antennas to cope with the isolation of the receiver from the transmitter, we may consider a "distributed version" of such approach where the transmit and receive antennas belong to physically separated nodes. This has the advantage that each of such nodes operates in conventional half-duplex mode. Furthermore, by allowing a large physical separation between nodes, the problem of receiver power saturation is eliminated.

Motivated by the distributed approach, we introduce a communication scheme that utilizes "virtual" full-duplex relays, each consisting of two half-duplex relays. In this configuration, each relay stage is formed of at least two half-duplex relays, used alternatively in transmit and receive modes, such that while one relay transmits its signal to the next stage, the other relay receives a signal from the previous stage. The role of the relays is swapped at the end of each time interval

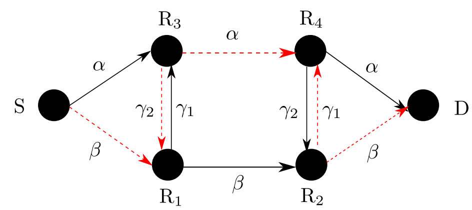

Fig. 1. Multihop virtual full-duplex relay channels when $K = 2$ (i.e., 3-hop relay network). Black solid-lines are active for every odd time slot and red-dashed lines are active for every even time slot.

(see Fig. 1). This relaying operation is known as "successive relaying" [2]. In this way, the source can send a new message to the destination at every time slot as if full-duplex relays are used. Every two consecutive source messages will travel via two alternate disjoint paths of relays. In [3], 2-hop model has been studied, showing that dirty paper coding (DPC) achieves the performance of *ideal* full-duplex relay since the source can completely eliminate the "known" interference at intended receiver. However, DPC is no longer applicable in a multihop network model shown in Fig. 1 since a transmit relay has no knowledge of interference signals at other stages. Thus, finding an optimal strategy for the multihop models is still an open problem.

In this paper, we present a coding scheme using the principle of QMF that does not require "long" messages and joint (non-unique) decoding as in [4], [5]. In particular, the destination explicitly decodes all relays' messages (i.e., bin-indices) and exploits them as side-information in the next time slot. This enables us to find an optimal quantization at each relay. Using this scheme, we show that the performance of this scheme degrades only *logarithmically* with the number of relay stages K as $\log(K)$, instead of linearly as in the case of "background noise-level" quantization [4], [5]. Furthermore, we propose a *mixed* strategy where some of relays perform DF and other relays perform QMF. As proposed in [6], QMF relays use rate-splitting so that DF relays can partially cancel the inter-relay interference. We show that this strategy can further reduce the performance degradation by a factor of two for ceratin channel conditions.

II. Network Models

We consider a virtual full-duplex relay channel with K relay stages illustrated in Fig. 1. Encoding/decoding operations are performed over time slots consisting of n channel uses of a discrete-time Gaussian channel. *Successive relaying* is assumed such that, at each time slot t, the source transmits a new message $\underline{\mathbf{w}}_t \in \{1, \ldots, 2^{nr_i}\}$ where $i = 1$ for odd time slot t and $i = 2$ for even time slot t, and the destination decodes a new message $\underline{\mathbf{w}}_{t-K}$. We define two message rates r_1 and r_2 since the odd-indexed and even-indexed messages are conveyed to the destination via two disjoint paths, namely, path 1: $(\mathrm{S}, \mathrm{R}_3, \mathrm{R}_4, \mathrm{D})$ and path 2: $(\mathrm{S}, \mathrm{R}_1, \mathrm{R}_2, \mathrm{D})$ in Fig. 1. The roles of relays alternate in successive time slots (see Fig. 1). During $N + K$ time slots, the destination decodes $N/2$ messages from each path. Thus, the achievable rate of the messages vis path i is given by $r_i N/2(N + K)$. By letting $N \to \infty$, the rate $r_i/2$ is achievable, provided that the message error probability vanishes with n. As in standard relay channels (see for example [4]–[7]), we take first the limit for $n \to \infty$ and then for $N \to \infty$, and focus on the achievable rate r_i. A block of n channel uses of the discrete-time channel in Fig. 1 is described by[1]

- For odd time slot t, relays 2, 3, and destination receive

$$\begin{aligned}
\underline{\mathbf{y}}_{\mathrm{R}_2}[t] &= \beta^2 \underline{\mathbf{x}}_{\mathrm{R}_1}[t] + \gamma_2^2 \underline{\mathbf{x}}_{\mathrm{R}_4}[t] + \underline{\mathbf{z}}_{\mathrm{R}_2}[t] \\
\underline{\mathbf{y}}_{\mathrm{R}_3}[t] &= \alpha^2 \underline{\mathbf{x}}_{\mathrm{S}}[t] + \gamma_1^2 \underline{\mathbf{x}}_{\mathrm{R}_1}[t] + \underline{\mathbf{z}}_{\mathrm{R}_3}[t] \\
\underline{\mathbf{y}}_{\mathrm{R}_D}[t] &= \alpha^2 \underline{\mathbf{x}}_{\mathrm{R}_4}[t] + \underline{\mathbf{z}}_{\mathrm{D}}[t]
\end{aligned}$$

- For even time slot t, relays 1, 4, and destination receive

$$\begin{aligned}
\underline{\mathbf{y}}_{\mathrm{R}_1}[t] &= \beta^2 \underline{\mathbf{x}}_{\mathrm{S}}[t] + \gamma_2^2 \underline{\mathbf{x}}_{\mathrm{R}_3}[t] + \underline{\mathbf{z}}_{\mathrm{R}_1}[t] \\
\underline{\mathbf{y}}_{\mathrm{R}_4}[t] &= \alpha^2 \underline{\mathbf{x}}_{\mathrm{R}_3}[t] + \gamma_1^2 \underline{\mathbf{x}}_{\mathrm{R}_2}[t] + \underline{\mathbf{z}}_{\mathrm{R}_4}[t] \\
\underline{\mathbf{y}}_{\mathrm{R}_D}[t] &= \beta^2 \underline{\mathbf{x}}_{\mathrm{R}_2}[t] + \underline{\mathbf{z}}_{\mathrm{D}}[t]
\end{aligned}$$

where $\alpha, \beta, \gamma_1, \gamma_2 \in \mathbb{R}_+$ denote channel gains. Here, $\underline{\mathbf{x}}_{\mathrm{S}}[t] \in \mathbb{C}^{1\times n}$ and $\underline{\mathbf{x}}_{\mathrm{R}_k}[t] \in \mathbb{C}^{1\times n}$ denote respective transmit signals at source and relay k, and $\underline{\mathbf{y}}_{\mathrm{D}}[t] \in \mathbb{C}^{1\times n}$ and $\underline{\mathbf{y}}_{\mathrm{R}_k}[t] \in \mathbb{C}^{1\times n}$ denote the respective received signals at the destination and relay k. Here we assume that each receiver can only receive signals from relays in the previous and the current relay stages, which is typically the case in wireless backhaul as the transmitted signal is often beamformed towards one direction. All noise processes are assumed to be i.i.d. Gaussian with zero mean and unit variance. Power constraint at each transmitter is given by SNR.

III. Main Results

The multihop virtual full-duplex relay channel shown in Fig. 1 is a special case of a general multiple multicast relay network analyzed in [4] and later on in [5] for a larger class of relay networks. In [4], QMF coding scheme was proposed and extended in [5] as NNC. NCC consists of message repetition encoding (i.e., one long message with repetitive encoding), signal quantization at relays, and simultaneous joint decoding (JD) of the received signals from all the blocks without explicitly decoding quantization indices. Recently, short-message NNC (SNNC) has been proposed in [7], which overcomes the long delay of NNC, by transmitting a new short

[1]This description can be straightforwardly extended into a K-stage network.

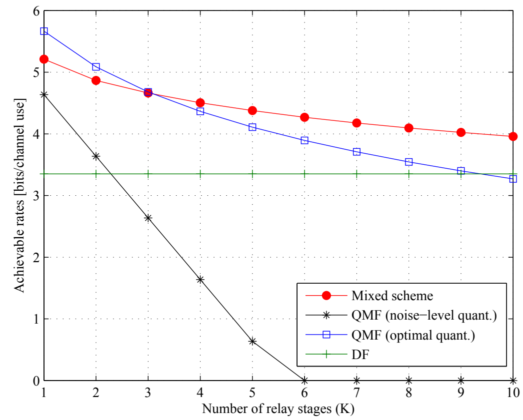

Fig. 2. The 3-hop full-duplex relay channels with asymmetric strength of inter-path interference where $\alpha^2\mathsf{SNR} = \beta^2\mathsf{SNR} = 20$ dB, $\gamma_1^2\mathsf{SNR} = 5$ dB, and $\gamma_2^2\mathsf{SNR} = 20$ dB.

message in each block, rather than the same message in each block. By setting the quantization distortion level to be at background noise level, QMF, NNC, and SNNC achieve the capacity within a constant gap where the gap scales *linearly* with the number of nodes, independently of SNR. For the channel considered in this paper, the achievable rate of QMF with noise level quantization in K-stage network is given by

$$R^{(K)}_{\mathrm{QMF-N}} = \frac{1}{2}\left(\log(1 + \alpha^2\mathsf{SNR}) + \log(1 + \beta^2\mathsf{SNR})\right) - K.$$

The detailed proof is omitted due to the space limit (see Lemma 5 in the longer version of this paper [9]). Accordingly, the performance gap scales *linearly* as

$$R^{(1)}_{\mathrm{QMF-N}} - R^{(K)}_{\mathrm{QMF-N}} = K - 1. \tag{1}$$

In this paper, we obtain an improved result to the capacity scaling as given in Corollary 1. By using an optimal quantization at the relays, we show that the gap scales *logarithmically* as $\log(K)$. Furthermore, this is achieved with a lower decoding complexity at destination than in NNC (or QMF with noise level quantization), since *successive decoding* (SD) is used instead of JD. Specifically, the destination *successively* decodes all relays' messages (i.e., quantization indices) and then the source message. Notice that all relays' messages are explicitly decoded, differently from NNC (and same as in SNNC) and are used as a side-information in the next time slot. Due to the use of Wyner-Ziv quantization[2], the destination can perform SD thereby being able to avoid the complexity of JD in order to decode each relay's message. Namely, the destination first finds a unique quantized sequence using the side-information and the bin-index, and then decodes the source message from the quantized observation (see Section IV-A). Before stating our main results, we give the following definition:

$$\mathcal{C}(x, K) \triangleq \log\left((1 + x)^{K+1} / \left((1 + x)^{K+1} - x^{K+1}\right)\right).$$

[2]We proved that Wyner-Ziv quantization is optimal for this model (see [9] for the detailed proof.)

Then, we have the following:

Theorem 1: For the $(K+1)$-hop virtual full-duplex relay channel shown in Fig. 1, the achievable rate region of QMF with *optimal* quantization is the set of all rate pairs $(r_1/2, r_2/2)$ such that

$$r_1 \leq \mathcal{C}(\alpha^2\mathsf{SNR}, K) \text{ and } r_2 \leq \mathcal{C}(\beta^2\mathsf{SNR}, K).$$

Proof: See Section IV-A. ■

Remark 1: We observe that the above achievable rate is independent of inter-relay interference levels γ_1 and γ_2. This is because the destination can completely eliminate the inter-relay interferences by exploiting the relays' messages decoded in the previous time slot as a side-information (see Section IV-A).

Let $R^{(K)}_{\text{QMF}} = (r_1 + r_2)/2$. We have the following:

Corollary 1: The performance degradation of QMF with optimal quantization with the number of relay stages K is

$$R^{(1)}_{\text{QMF}} - R^{(K)}_{\text{QMF}} \leq \log(K+1) - 1. \tag{2}$$

Proof: From Theorem 1 and with $\alpha = \beta = 1$, we can compute

$$\begin{aligned} R^{(1)}_{\text{QMF}} - R^{(K)}_{\text{QMF}} &= \log\left(\frac{\sum_{i=0}^{K}(1+\mathsf{SNR})^{K-i}\mathsf{SNR}^i}{(1+\mathsf{SNR})^{K-1}(1+2\mathsf{SNR})}\right) \\ &\leq \log\left(\frac{\mathsf{SNR}\sum_{i=0}^{K}(1+\mathsf{SNR})^{K-1}}{(1+\mathsf{SNR})^{K-1}(1+2\mathsf{SNR})}\right) \\ &= \log(K+1) + \log\left(\mathsf{SNR}/(1+2\mathsf{SNR})\right) \\ &\leq \log(K+1) - 1. \end{aligned}$$

Since the gap is independent of SNR (i.e., α and β), this completes the proof. ■

As pointed out in [7], an advantage of using a short message (as in SNNC and in the proposed scheme) is that it allows some of relays to decode a source message and forward it downstream. Furthermore, it was demonstrated in [7] that a mixed strategy that allows some of relays to perform DF while using SNNC at the rest can outperform NNC (or SNNC). This is because the relays in favorable positions can decode a source message without rate penalty and eliminate the noise that is otherwise propagated via quantization based scheme. Motivated by this, we also consider a mixed strategy where in Fig. 1, the relays in the top path perform DF and relays in the bottom path perform QMF. In [7], DF relays decode a desired message by treating other signals as noise, which can degrade the performance especially when the interference is strong. To overcome this problem, we will use *rate-splitting* that allows DF relays to partially decode the interference signal created by QMF relays. The idea is to incorporate a superposition coding [8] into QMF encoding, i.e., the bin indices at QMF relays are encoded using rate-splitting followed by superposition coding as done in [6]. Using this scheme, we obtain the following:

Theorem 2: For a $(K+1)$-hop virtual full-duplex relay channel shown in Fig. 1, the achievable rate region of mixed scheme is the set of all rate pairs $(r_1/2, r_2/2)$ such that

$$\begin{aligned} r_1 &\leq \log\left(1 + \alpha^2\mathsf{SNR}/(1+\theta\gamma_1^2\mathsf{SNR})\right) \\ r_2 &\leq \log(1 + \beta^2\mathsf{SNR}/(1+\hat{\sigma}_1^2)) \end{aligned}$$

for some power-splitting parameter $\theta \in [0,1]$, where $\hat{\sigma}_1^2$ is computed by the following recursive equation:

$$\hat{\sigma}_k^2 = \max\left\{(1+\hat{\sigma}_{k+1}^2)\frac{1+\beta^2\mathsf{SNR}}{\beta^2\mathsf{SNR}}, \frac{1+\beta^2\mathsf{SNR}}{\phi_k - 1}\right\},$$

for $k = K, \ldots, 1$ and with initial value $\hat{\sigma}_{K+1}^2 = 0$, where

$$\phi_k = \left(1 + \frac{\theta\beta^2\mathsf{SNR}}{1+\hat{\sigma}_{k+1}^2}\right)\left(1 + \frac{(1-\theta)\gamma_1^2\mathsf{SNR}}{1+\alpha^2\mathsf{SNR}+\theta\gamma_1^2\mathsf{SNR}}\right).$$

Proof: See Section IV-B. ■

Let $R^{(K)}_{\text{QMF-DF}} = (r_1 + r_2)/2$. We have the following:

Corollary 2: Performance degradation of the mixed QMF with rate splitting and DF strategy, with the number of relay stages K is

$$R^{(1)}_{\text{QMF-DF}} - R^{(K)}_{\text{QMF-DF}} \leq \log(\sqrt{K}). \tag{3}$$

Proof: The proof follows from Theorem 2. ■

Remark 2: Since $R^{(1)}_{\text{QMF}}$ is within 1 bit gap from the cut-set upper bound for any SNR, QMF with optimal quantization is within $\log(K)$ gap from the cut-set upper bound for K-stage network. For $R^{(1)}_{\text{QMF-DF}}$, however, the gap from the capacity depends on SNR and channel coefficients. Thus, the mixed scheme can achieve the capacity within $\log(\sqrt{K})$ gap for certain channel coefficients (as in Fig. 2).

Example 1: We numerically evaluate achievable average rates $(r_1+r_2)/2$ of the proposed schemes for different values of K, in Fig. 2. We also compare it to DF-only scheme (i.e., all relays perform DF) for which the achievable rate of DF is derived in [9], is independent of K, and is obtained by each relay either treating interference as noise (TIN) or JD depending on the interference level. In this example, JD achieves a higher rate than TIN. We observe that the capacity scaling plays an important role when considering a multihop transmission and operating at finite SNR. Furthermore, we emphasize that the proposed schemes (optimized QMF or mixed scheme) outperform DF-only scheme and have lower decoding complexity since the former performs SD while the latter (in this example) performs JD.

IV. Proofs

In this section, we prove our main results stated in Section III by presenting two coding schemes.

A. Proof of Theorem 1: Optimized QMF scheme

To derive the achievable rate expression for QMF (with optimal quantization), we consider the *time-expanded* network for 3-hop virtual full-duplex relay network in Fig. 3. Focusing on decoding $\underline{\mathbf{w}}_3$, we can produce the simplified channel model illustrated in Fig. 4. This model can be applied to the decoding of any source message $\underline{\mathbf{w}}_t$ for $t \geq 3$, where the channel gains are replaced by α and γ_1 for odd-indexed messages. Hence, we

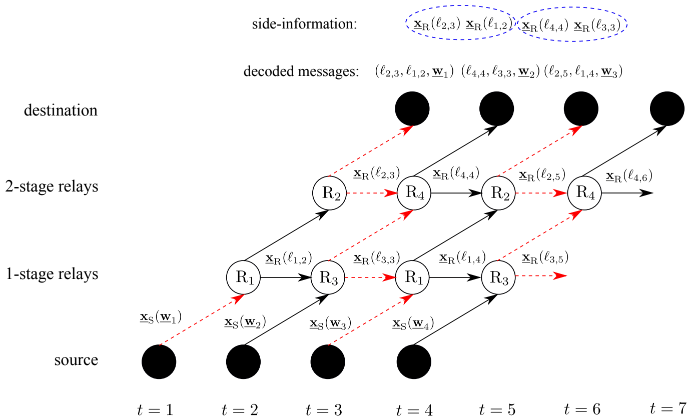

Fig. 3. Time expanded 3-hop network where $\ell_{k,t}$ denotes the relay k's message at time slot t.

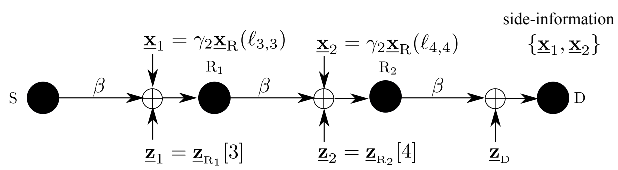

Fig. 4. Equivalent model of QMF scheme for 3-hop network.

can drop the time index and assume $\alpha = \beta = 1$, $\gamma_1 = \gamma_2 = \gamma$, and $r_1 = r_2 = r$ in the simplified model. Also, we follow the notation in Fig. 4 for the "known" interferences and the additive Gaussian noises.

The simplified model in Fig. 3 can be easily extended to $(K+1)$-hop network with the following notations. Let $\underline{\mathbf{x}}_{\rm R}(\ell_k)$ denote the transmit signal of relay k with message ℓ_k, for $k = 1, \ldots, K$. Also, let r_k denote the message rate of relay k (i.e., $\ell_k \in \{1, \ldots, 2^{nr_k}\}$). Letting $\underline{\mathbf{y}}_k$ denote the received signal at relay k, we have:

$$\underline{\mathbf{y}}_k = \underline{\mathbf{x}}_{\rm R}(\ell_{k-1}) + \underline{\mathbf{x}}_k + \underline{\mathbf{z}}_k, k = 1, \ldots, K, \tag{4}$$

where $\underline{\mathbf{z}}_k$ consists of an i.i.d. complex Gaussian random variable with zero mean and variance 1. The received signal at destination is given by

$$\underline{\mathbf{y}}_{\rm D} = \underline{\mathbf{x}}_{\rm R}(\ell_K) + \underline{\mathbf{z}}_{\rm D}, \tag{5}$$

where $\underline{\mathbf{z}}_{\rm D}$ consists of an i.i.d. complex Gaussian random variable with zero mean and unit variance.

The procedures of encoding/decoding are as follows: Each relay k uses a quantization codebook $\{\underline{\dot{\mathbf{y}}}_k(1), \ldots, \underline{\dot{\mathbf{y}}}_k(2^{nR_0})\}$ of block length n, generated at random with i.i.d. components according to the distribution of $\dot{Y}_k = Y_k + \dot{Z}_k$ where $\dot{Z}_k \sim \mathcal{CN}(0, \hat{\sigma}_k^2)$. Notice that R_0 is completely determined by the quantization distortion level $\hat{\sigma}_k^2$. The quantization codebook is partitioned into $2^{n(R_k - \delta)}$ bins, by a random assignment, such that each bin has size $2^{n(R_0 - r_k + \delta)}$, for some $\delta > 0$.

Encoding:

- Source encodes a message $\underline{\mathbf{w}} \in \{1, \ldots, 2^{nr}\}$ and transmits $\underline{\mathbf{x}}_{\rm S}(\underline{\mathbf{w}})$ to relay 1.
- Relay k quantizes its received signal $\underline{\mathbf{y}}_k = \underline{\mathbf{x}}_{\rm R}(\ell_{k-1}) + \underline{\mathbf{x}}_k + \underline{\mathbf{z}}_k$ into a quantization codeword $\underline{\dot{\mathbf{y}}}_k$.
- The relay finds a bin index $\ell_k \in \{1, \ldots, 2^{nr_k}\}$ such that it contains the quantization codeword $\underline{\dot{\mathbf{y}}}_k$, it encodes the bin index as $\underline{\mathbf{x}}_{\rm R}(\ell_k)$, and transmits to the next stage relay $k+1$. Here, Wyner-Ziv quantization [8] is used, such that the quantization distortion level is chosen by imposing

$$r_k = I(Y_k; \dot{Y}_k | X_k) = \log\left(1 + (1 + \mathsf{SNR})/\hat{\sigma}_k^2\right). \tag{6}$$

Decoding at the destination:

- From the received signal $\underline{\mathbf{y}}_{\rm D}$, the destination can decode the bin-index ℓ_K if

$$r_K \leq \log(1 + \mathsf{SNR}). \tag{7}$$

- Using the decoded bin-index ℓ_K and side-information $\underline{\mathbf{x}}_K$, it finds an unique quantization codeword $\underline{\dot{\mathbf{y}}}_K = \underline{\mathbf{x}}_{\rm R}(\ell_{K-1}) + \underline{\mathbf{x}}_K + \underline{\mathbf{z}}_K + \underline{\dot{\mathbf{z}}}_K$, from which the known interference $\underline{\mathbf{x}}_K$ can be canceled, obtaining

$$\underline{\dot{\mathbf{y}}}_K - \underline{\mathbf{x}}_K = \underline{\mathbf{x}}_{\rm R}(\ell_{K-1}) + \underline{\mathbf{z}}_K + \underline{\dot{\mathbf{z}}}_K. \tag{8}$$

Then, the destination can decode the bin index ℓ_{K-1} if

$$r_{K-1} \leq \log\left(1 + \mathsf{SNR}/(1 + \hat{\sigma}_K^2)\right). \tag{9}$$

Following this procedure, the destination can reliably decode the bin index ℓ_k if for $k = 1, \ldots, K-1$

$$r_k \leq \log\left(1 + \mathsf{SNR}/(1 + \hat{\sigma}_{k+1}^2)\right). \tag{10}$$

- By repeating the above procedure until all relay bin indices have been decoded, the destination obtains:

$$\underline{\dot{\mathbf{y}}}_1 = \underline{\mathbf{x}}_{\rm S}(\underline{\mathbf{w}}) + \underline{\mathbf{z}}_1 + \underline{\dot{\mathbf{z}}}_1. \tag{11}$$

- Finally, destination can decode the source message $\underline{\mathbf{w}}$ if

$$r \leq \log\left(1 + (\mathsf{SNR})/(1 + \hat{\sigma}_1^2)\right). \tag{12}$$

In order to derive an achievable rate, we need to find $\hat{\sigma}_1^2$ by considering all rate constraints in (10). First, we choose r_K defined in (6) such that the rate-constraint of (7) is satisfied with equality. Then, we have:

$$\log\left(1 + (1 + \mathsf{SNR})/\hat{\sigma}_K^2\right) = \log(1 + \mathsf{SNR}),$$

which yields $\hat{\sigma}_K^2 = (1 + \mathsf{SNR})/\mathsf{SNR}$. Similarly, using (6) and from the rate constraint in (9), we have:

$$\log\left(1 + (1 + \mathsf{SNR})/(\hat{\sigma}_{K-1}^2)\right) = \log\left(1 + \mathsf{SNR}/(1 + \hat{\sigma}_K^2)\right),$$

which yields $\hat{\sigma}_{K-1}^2 = ((1 + \mathsf{SNR})/\mathsf{SNR})(1 + \hat{\sigma}_K^2)$. In general, we have the following relation:

$$\hat{\sigma}_{k-1}^2 = ((1 + \mathsf{SNR})/\mathsf{SNR})(1 + \hat{\sigma}_k^2), \tag{13}$$

for $k = K, K-1, \ldots, 2$, where initial value $\hat{\sigma}_K^2 = (1 + \mathsf{SNR})/\mathsf{SNR}$. Using this relation, we find

$$\begin{aligned}\hat{\sigma}_1^2 &= \sum_{i=1}^{K} ((1 + \mathsf{SNR})/\mathsf{SNR})^i \\ &= ((1 + \mathsf{SNR})^{K+1} - (1 + \mathsf{SNR})\mathsf{SNR}^K)/\mathsf{SNR}^K. \end{aligned} \tag{14}$$

By substituting (14) into (12), the achievable rate of QMF for the $(K+1)$-hop network is given by

$$r = \log\left((1+\mathsf{SNR})^{K+1}/((1+\mathsf{SNR})^{K+1}-\mathsf{SNR}^{K+1})\right),$$

which is equal to $\mathcal{C}(\mathsf{SNR},K)$. Using the above result in our original channel model, we immediately obtain the following rate constraints:

$$r_1 \le \mathcal{C}(\alpha^2\mathsf{SNR},K) \text{ and } r_2 \le \mathcal{C}(\beta^2\mathsf{SNR},K).$$

This completes the proof.

B. Proof of Theorem 2: Mixed scheme

We let $\underline{\mathbf{x}}_k$ and $\underline{\mathbf{y}}_k$ denote the transmit and receive signals of relay k for $k=1,\ldots,K$. In Fig. 1, relays indexed by $k=1,\ldots,K$ perform QMF as proposed in Section IV-A and relays indexed by $k=K+1,\ldots,2K$ perform DF. Furthermore, QMF relays use rate-splitting so that DF relays can partially cancel the inter-relay interference. To be specific, each QMF relay k finds a bin index ℓ_k using the encoding procedures in Section IV-A. Then, split message ℓ_k into independent "common" message $\ell_{k,c} \in \{1,\ldots,2^{nr_{k,c}}\}$ and "private" message $\ell_{k,p} \in \{1,\ldots,2^{nr_{k,p}}\}$ with $r_k = r_{k,c}+r_{k,p}$. Here, r_k is chosen as in (6) using Wyner-Ziv quantization:

$$r_{k,c}+r_{k,p} = \log\left(1+(1+\mathsf{SNR})/\hat{\sigma}_k^2\right). \quad (15)$$

The relay encodes its message $(\ell_{k,c},\ell_{k,p})$ using superposition coding such as

$$\underline{\mathbf{x}}_k(\ell_{k,c},\ell_{k,p}) = (1-\theta)\underline{\mathbf{x}}_{k,c}(\ell_{k,c}) + \theta\underline{\mathbf{x}}_{k,p}(\ell_{k,p}) \quad (16)$$

for some power-splitting parameter $\theta \in [0,1]$.

Decoding messages via QMF relays: Following the results in Section IV-A, the destination can reliably decode the relays's messages using SD from a quantized observation if

$$r_{k,c} \le \log\left(1+(1-\theta)\beta^2\mathsf{SNR}/(1+\hat{\sigma}_{k+1}^2+\theta\beta^2\mathsf{SNR})\right) \quad (17)$$

$$r_{k,p} \le \log\left(1+\theta\beta^2\mathsf{SNR}/(1+\hat{\sigma}_{k+1}^2)\right), \quad (18)$$

for $k=1,\ldots,K$, where $\hat{\sigma}_{K+1}^2 = 0$. The destination can reliably decode source messages if

$$r_2 \le \log\left(1+\beta^2\mathsf{SNR}/(1+\hat{\sigma}_1^2)\right). \quad (19)$$

Decoding messages via DF relays: We use the simplified model in Fig. 5. Each relay $K+k$ observes

$$\underline{\mathbf{y}}_{K+k} = \underline{\mathbf{x}}_{K+k-1}(\underline{\mathbf{w}}) + \gamma_1^2\underline{\mathbf{x}}_k + \underline{\mathbf{z}}_{K+k}, \quad (20)$$

where $\underline{\mathbf{x}}_k = (1-\theta)\underline{\mathbf{x}}_{k,c}(\ell_{k,c}) + \theta\underline{\mathbf{x}}_{k,p}(\ell_{k,p})$. Relay $K+k$ performs the SD [8] where it first decodes the part of QMF relay's message $\ell_{k,c}$, subtract $\underline{\mathbf{x}}_{k,c}(\ell_{k,c})$ from the received signal, and then decodes source message from the enhanced received signal. Using this scheme, it can decode source message $\underline{\mathbf{w}}$ if

$$r_{k,c} \le \log\left(1+(1-\theta)\gamma_1^2\mathsf{SNR}/(1+\theta\gamma_1^2\mathsf{SNR})\right) \quad (21)$$

$$r_1 \le \log\left(1+\alpha^2\mathsf{SNR}/(1+\theta\gamma_1^2\mathsf{SNR})\right). \quad (22)$$

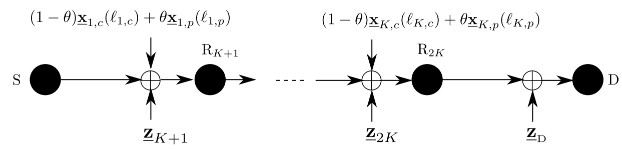

Fig. 5. Equivalent model of the mixed strategy for $(K+1)$-hop network.

Using Fourier-Motzkin elimination for (15), (17), and (21), we can eliminate $r_{k,c}$ and obtain

$$r_{k,p} \ge \log\left(1+(1+\beta^2\mathsf{SNR})/\hat{\sigma}_k^2\right) - \min\left\{\log\left(1+(1-\theta)\beta^2\mathsf{SNR}/(1+\theta\beta^2\mathsf{SNR}+\hat{\sigma}_{k+1}^2)\right), \log\left(1+(1-\theta)\gamma_1^2\mathsf{SNR}/(1+\alpha^2\mathsf{SNR}+\theta\gamma_1^2\mathsf{SNR})\right)\right\}. \quad (23)$$

Using (18) and (23), we can eliminate $r_{k,p}$ and obtain the equality constraint in Theorem 2. Together with (19) and (22), we complete the proof.

V. Concluding Remarks

We introduced a multihop "virtual" full-duplex relaying by means of half-duplex relays. By using the optimal quantization, it was shown that the gap from the capacity of QMF scales *logarithmically* with the number of relay stages K, instead of linearly as is the case of "noise level" quantization. We further improved the performance by presenting a mixed strategy where some of relays perform DF and other relays perform QMF with rate-splitting. Interested readers may refer to the longer version [9], where comparisons with various coding schemes such as amplify-and-forward and compute-and-forward are also provided. The multihop virtual relay network operation studied here can be applied, as a guiding principle, to the implementation of a wireless backhaul formed by multiple virtual full duplex relay stages.

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