Capacity of Interference Channels With Partial Transmitter Cooperation

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***Abstract*—Capacity regions are established for several two-sender, two-receiver channels with partial transmitter cooperation. First, the capacity regions are determined for compound multiple-access channels (MACs) with common information and compound MACs with conferencing. Next, two interference channel models are considered: an interference channel with common information (ICCI) and an interference channel with unidirectional cooperation (ICUC) in which the message sent by one of the encoders is known to the other encoder. The capacity regions of both of these channels are determined when there is strong interference, i.e., the interference is such that both receivers can decode all messages with no rate penalty. The resulting capacity regions coincide with the capacity region of the compound MAC with common information.**

***Index Terms*—Capacity region, cooperation, strong interference.**

I. Introduction

DISCRETE memoryless channels with two senders and two receivers permit various forms of sender cooperation. In the most restrictive circumstance when cooperation is precluded, we have the interference channel [1], [2]. However, cooperation among encoders can improve the achievable rates, as shown for Gaussian networks in [3]–[7] and multiple-access channels (MACs) with conferencing in [8]. In this paper, we examine two-sender, two-receiver communication systems that allow partial cooperation among the encoders by conferencing and signaling with common messages.

For a single receiver, the MAC with conferencing encoders has two communication links with finite capacities between the two encoders over which the encoders obtain partial information about each other's messages. This information is referred to as a *common* message as it is known to both encoders after conferencing. In addition, each encoder will still have independent information referred to as a *private* message, unknown to the other encoder. Consequently, the capacity region of the MAC with partially cooperating encoders is related to the capacity region of the MAC with common information. The capacity region of this latter channel was determined by Slepian and Wolf [9] (see also [10]).

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We use the approach of [9], [10] in Section II to establish the capacity region $\mathcal{C}_{\rm CMAC}$ of the compound MAC with common information (CMAC) that has *two* receivers decoding messages sent from both encoders. We then use this result in Section III to determine the capacity region of the compound MAC with conferencing encoders where the encoders communicate over separate links with finite capacities, as in [8]. In subsequent sections, two interference channel models with partial transmitter cooperation are considered. Specifically, in Section IV we relax the decoding constraint and assume that each decoder is interested only in a private message. In this case, the channel becomes an interference channel with common information (ICCI). We determine the capacity region of this channel for the special case of strong interference [11]–[13], i.e., the interference is such that both receivers can decode all messages. In Section V, we consider the interference channel with unidirectional cooperation (ICUC) in which the message at one of the encoders is made available to the other encoder. For the Gaussian case of weak interference, the capacity region of this channel was recently determined in [14], [15]. We derive capacity results for strong interference. For the interference channel, the capacity region in strong interference was determined in [13] and was shown to coincide with the capacity region of the two-sender, two-receiver compound MAC in which both messages are decoded at both receivers [16].

The four channel models considered here and the relationships between their capacity regions are shown in Fig. 1. In order to clarify these relationships, we introduce an indicator function

$$c_{\rm CMAC}(\bar{R}) = \begin{cases} 1, & \bar{R} \in \mathcal{C}_{\rm CMAC} \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

We similarly define the indicator functions $c(\cdot)$, $c_{\rm ICCI}(\cdot)$ and $c_{\rm ICUC}(\cdot)$ for the respective capacity regions of the compound MAC with conferencing, the ICCI and the ICUC. The results in this paper were presented in part in the conference papers [17]–[19].

II. The Compound MAC With Common Information

Consider a channel with finite-input alphabets $\mathcal{X}_1, \mathcal{X}_2$, finite-output alphabets $\mathcal{Y}_1, \mathcal{Y}_2$, and a conditional probability distribution $p(y_1, y_2|x_1, x_2)$, where $(x_1, x_2) \in \mathcal{X}_1 \times \mathcal{X}_2$ are channel inputs and $(y_1, y_2) \in \mathcal{Y}_1 \times \mathcal{Y}_2$ are channel outputs. Each encoder $t, t = 1, 2$, wishes to send a private message $W_t \in \{1, \ldots, M_t\}$ to decoders in N channel uses. In addition, a common message $W_0 \in \{1, \ldots, M_0\}$ is communicated from the encoders to both receivers. The channel is memoryless and time-invariant in the sense that

$$p\left(y_{1,n}, y_{2,n}|\boldsymbol{x}_1^n, \boldsymbol{x}_2^n, \boldsymbol{y}_1^{n-1}, \boldsymbol{y}_2^{n-1}, \bar{w}\right) = p_{Y_1,Y_2|X_1,X_2}(y_{1,n}, y_{2,n}|x_{1,n}, x_{2,n}) \quad (2)$$

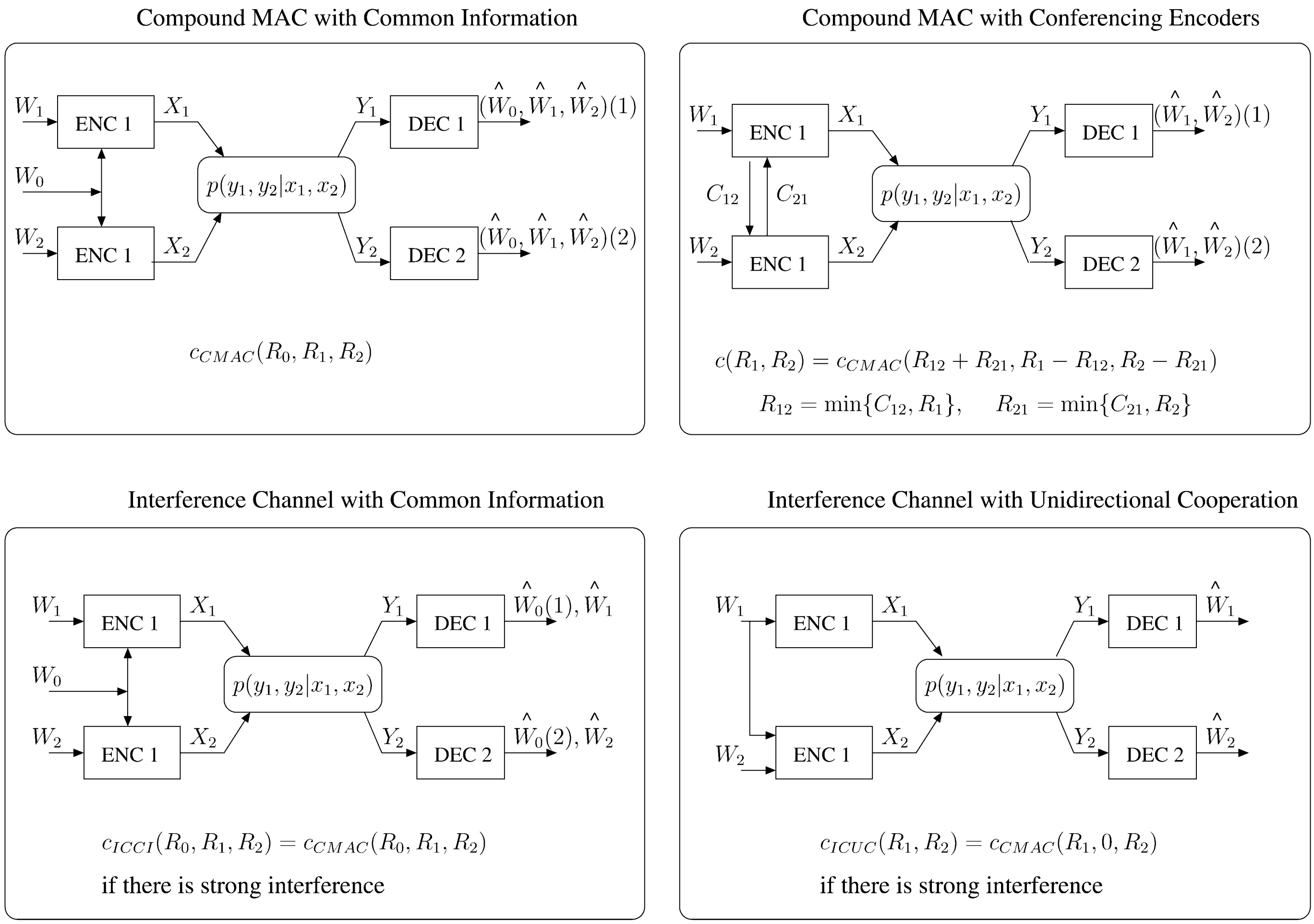

Fig. 1. Considered channel models with limited transmitter cooperation and the relationship between their capacity regions.

for all n, where X_1, X_2 and Y_1, Y_2 are random variables representing the respective inputs and outputs, $\bar{w} = [w_0, w_1, w_2]$ denotes the messages to be sent, and $\boldsymbol{x}_t^n = [x_{t,1}, \ \ldots, \ x_{t,n}]$. We will follow the convention of dropping subscripts of probability distributions if the arguments of the distributions are lower case versions of the corresponding random variables. To simplify notation, we also drop superscripts when $n = N$.

The messages W_0, W_1, and W_2 are independently generated at the beginning of each block of N channel uses. Encoder $t, t = 1, 2$, maps the common message W_0 and the private message W_t into a codeword

$$\boldsymbol{X}_t = f_t(W_0, W_t). \tag{3}$$

Decoder $t, t = 1, 2$, estimates W_0 and W_t based on the received N-sequence $\boldsymbol{Y}_t$ as

$$(\hat{W}_0(t), \hat{W}_1(t), \hat{W}_2(t)) = g_t(\boldsymbol{Y}_t). \tag{4}$$

An (M_0, M_1, M_2, N, P_e) code has two encoding functions f_1, f_2, two decoding functions g_1, g_2, and an error probability

$$P_e = \sum_{\bar{w}} \frac{1}{M_0 M_1 M_2} P\left[\{g_1(\boldsymbol{Y}_1) \neq \bar{w}\} \cup \{g_2(\boldsymbol{Y}_2) \neq \bar{w}\} \,|\bar{w} \text{ sent}\right]. \tag{5}$$

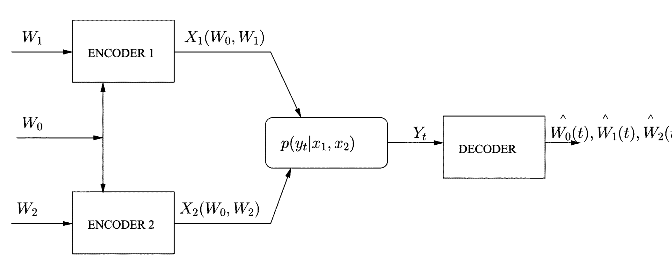

Fig. 2. MAC with common information.

A rate triple (R_0, R_1, R_2) is achievable if, for any $\epsilon > 0$, there is an (M_0, M_1, M_2, N, P_e) code such that

$$M_t \geq 2^{NR_t}, \qquad t = 0, 1, 2, \text{ and } P_e \leq \epsilon.$$

The capacity region $\mathcal{C}_{\text{CMAC}}$ of the compound MAC with common information is the closure of the set of all achievable rate triples (R_0, R_1, R_2). We next determine $\mathcal{C}_{\text{CMAC}}$ using a result of Slepian and Wolf [9].

The above channel becomes the MAC with Common Information if there is only one receiver (see Fig. 2). Consider the channel output Y_t. A code for this channel has two encoding functions (3), one decoding function (4), and an error probability

$$P_{e,t} = \sum_{\bar{w}} \frac{1}{M_0 M_1 M_2} P\left[g_t(\boldsymbol{Y}_t) \neq \bar{w} | \bar{w} \text{ sent}\right]. \tag{6}$$

To express the capacity region of the MAC with common information, we define

$$\begin{aligned}&\mathcal{R}_{\mathrm{MAC},t}(p(u),p(x_1|u),p(x_2|u))\\&=\Big\{(R_0,R_1,R_2):R_0\geq 0,R_1\geq 0,R_2\geq 0,\\&\quad R_1\leq I(X_1;Y_t|X_2,U)\\&\quad R_2\leq I(X_2;Y_t|X_1,U)\\&\quad R_1+R_2\leq I(X_1,X_2;Y_t|U)\\&\quad R_0+R_1+R_2\leq I(X_1,X_2;Y_t)\Big\}.\end{aligned}\tag{7}$$

The capacity region of the MAC with common information is

$$\mathcal{C}_{\mathrm{MAC},t}=\bigcup\mathcal{R}_{\mathrm{MAC},t}\left(p(u),p(x_1|u),p(x_2|u)\right)\tag{8}$$

where the union is over all joint distributions that factor as

$$p(u,x_1,x_2,y_t)=p(u)p(x_1|u)p(x_2|u)p(y_t|x_1,x_2).\tag{9}$$

We remark that the convex hull used in [9] was shown to be unnecessary in [10].

We use (8) to determine the capacity region of the compound MAC with common information. Observe that this channel $(\mathcal{X}_1\times\mathcal{X}_2,p(y_1,y_2|x_1,x_2),\mathcal{Y}_1\times\mathcal{Y}_2)$ defines two MACs

$$p(y_1|x_1,x_2)=\sum_{y_2\in\mathcal{Y}_2}p(y_1,y_2|x_1,x_2)\tag{10}$$

$$p(y_2|x_1,x_2)=\sum_{y_1\in\mathcal{Y}_1}p(y_1,y_2|x_1,x_2).\tag{11}$$

We adapt the coding strategy of Willems in [10] to prove the following result.

Theorem 1: The capacity region of the compound MAC with common information is

$$\begin{aligned}&\mathcal{C}_{\mathrm{CMAC}}\\&=\bigcup\Big\{\mathcal{R}_{\mathrm{MAC},1}\bigcap\mathcal{R}_{\mathrm{MAC},2}\Big\}\\&=\bigcup\{(R_0,R_1,R_2):R_0\geq 0,R_1\geq 0,R_2\geq 0,\\&\quad R_1\leq\min\{I(X_1;Y_1|X_2,U),I(X_1;Y_2|X_2,U)\}\\&\quad R_2\leq\min\{I(X_2;Y_1|X_1,U),I(X_2;Y_2|X_1,U)\}\\&\quad R_1+R_2\leq\min\{I(X_1,X_2;Y_1|U),I(X_1,X_2;Y_2|U)\}\\&\quad R_0+R_1+R_2\leq\min\{I(X_1,X_2;Y_1),I(X_1,X_2;Y_2)\Big\}\end{aligned}\tag{12}$$

where the union is over all joint distributions that factor as (9) for $t=1,2$.

Proof: For the converse, consider an (M_0,M_1,M_2,N,P_e) code for a CMAC. From [10, Sec. 3.4] it follows that for $t=1,2$, (R_0,R_1,R_2) belongs to

$$\begin{aligned}\mathcal{R}_t^N=\Big\{&(R_0,R_1,R_2):R_0\geq 0,R_1\geq 0,R_2\geq 0,\\&R_1\leq\frac{1}{N}\sum_{n=1}^{N}I(X_{1n};Y_{tn}|X_{2n},W_0)+\epsilon(P_{e,t})\\&R_2\leq\frac{1}{N}\sum_{n=1}^{N}I(X_{2n};Y_{tn}|X_{1n},W_0)+\epsilon(P_{e,t})\\&R_1+R_2\leq\frac{1}{N}\sum_{n=1}^{N}I(X_{1n},X_{2n};Y_{tn}|W_0)+\epsilon(P_{e,t})\\&R_0+R_1+R_2\leq\frac{1}{N}\sum_{n=1}^{N}I(X_{1n},X_{2n};Y_{tn})+\epsilon(P_{e,t})\Big\}.\end{aligned}\tag{13}$$

where

$$p(x_{1n},x_{2n}|w_0)=p(x_{1n}|w_0)p(x_{2n}|w_0)\tag{14}$$

and

$$\epsilon(P_{e,t})\to 0,\qquad\text{as } P_e\to 0.\tag{15}$$

Continuing as in [10, Sec. 3.4], the region $\mathcal{R}_t^N$ satisfies

$$\begin{aligned}\mathcal{R}_t^N\subseteq\Big\{&(R_0,R_1,R_2):R_0\geq 0,R_1\geq 0,R_2\geq 0,\\&R_1\leq I(X_1;Y_t|X_2,U)+\epsilon(P_{e,t})\\&R_2\leq I(X_2;Y_t|X_1,U)+\epsilon(P_{e,t})\\&R_1+R_2\leq I(X_1,X_2;Y_t|U)+\epsilon(P_{e,t})\\&R_0+R_1+R_2\leq I(X_1,X_2;Y_t)+\epsilon(P_{e,t})\Big\}\end{aligned}\tag{16}$$

where $U=W_0$ and, using $U_n=W_0$, we have

$$\begin{aligned}&P_{UX_1X_2Y_t}(a,b,c,d)\\&=\frac{1}{N}\sum_{n=1}^{N}P_{U_n}(a)P_{X_{1n}|U_n}(b|a)P_{X_{2n}|U_n}(c|a)P_{Y_t|X_1X_2}(d|b,c).\end{aligned}\tag{17}$$

From (13), (15)–(17), and by comparing (16) with (7), we observe that

$$\begin{aligned}(R_0,R_1,R_2)\in\mathcal{R}_{\mathrm{MAC},1}&\left(p(u),p(x_1|u),p(x_2|u)\right)\\&\bigcap\mathcal{R}_{\mathrm{MAC},2}\left(p(u),p(x_1|u),p(x_2|u)\right)\end{aligned}$$

where $p(u,x_1,x_2,y_t)$ is computed via (17). This completes the converse.

For achievability, we adapt the encoding and decoding strategy proposed by Willems in [10] to achieve the rates (12). Specifically, we use the codebook in [10, Sec. 3] constructed as follows.

1) Fix the distribution $p(u,x_1,x_2)=p(u)p(x_1|u)p(x_2|u)$.
2) Generate M_0 sequences $\boldsymbol{u}$ each with probability

$$p(\boldsymbol{u})=\prod_{n=1}^{N}p_U(u_n).$$

Label these sequences $\boldsymbol{u}(w_0)$, $w_0\in\{1,\ldots,M_0\}$.

3) For each $\boldsymbol{u}(w_0)$, generate M_t sequences $\boldsymbol{x}_t$ with probability

$$P(\boldsymbol{x}_t|\boldsymbol{u})=\prod_{n=1}^{N}p_{X_t|U}(x_{tn}|u_n)$$

where $t=1,2$. Label them $\boldsymbol{x}_t(w_0,w_t)$, $w_t\in\{1,\ldots,M_t\}$.

Encoding: To send a common message w_0 and a private message w_t encoder t sends the codeword $\boldsymbol{x}_t(w_0,w_t)$.

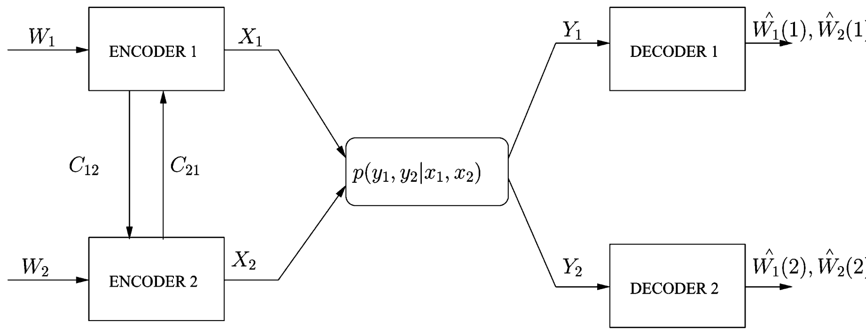

Fig. 3. Compound MAC with conferencing encoders.

Decoding: At each decoder, we use the decoding scheme of [10]. After receiving $\boldsymbol{y}_t$, decoder t tries to find a $(\hat{w}_0, \hat{w}_1, \hat{w}_2)$ such that

$$(\boldsymbol{u}(\hat{w}_0), \boldsymbol{x}_1(\hat{w}_0, \hat{w}_1), \boldsymbol{x}_2(\hat{w}_0, \hat{w}_2), y_t) \in A_\epsilon(P_{UX_1X_2Y_t})$$

where $A_\epsilon(P_{UX_1X_2Y_t})$ is the set of ϵ-typical N-sequences $(\boldsymbol{u}, \boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{y}_t)$ with respect to the distribution (9), as defined in [20, Sec. 14.2].

Error Probability: We apply the union bound to (5) to obtain

$$P_e \leq P_{e,1} + P_{e,2} \tag{18}$$

where $P_{e,1}$ and $P_{e,2}$ are given by (6). It was shown in [10] that $P_{e,1}$ and $P_{e,2}$ can be made arbitrarily close to zero when the rates satisfy (12). From (18) it then follows that P_e can be made arbitrarily close to zero. □

Observe from (12) that $\mathcal{C}_{\mathrm{CMAC}}$ depends only on the marginal distributions $p(y_t|x_1, x_2)$. Further, one can show that $\mathcal{C}_{\mathrm{CMAC}}$ is convex by using the proof technique of [10, Appendix A].

III. The Capacity Region of the Compound MAC With Conferencing

Suppose there are two links with finite capacities C_{12} and C_{21} between the two encoders, as shown in Fig. 3. We refer to this channel as a compound MAC with conferencing encoders. The model is the same as in Section II except that the encoders use their communication links in the form of a *conference* [8]. A conference has two sets of K functions $\{h_{t,1}, \ldots, h_{t,K}\}$, $t = 1, 2$. Each function $h_{t,k}$ maps the message W_t and the sequence of previously received symbols from the other encoder into the kth symbol $V_{t,k}$, where $V_{t,k}$ has the finite alphabet $\mathcal{V}_{t,k}$, $k = 1, \ldots, K$. We write this as

$$h_{1,k} : \mathcal{W}_1 \times \mathcal{V}_2^{k-1} \to \mathcal{V}_{1,k}, \quad V_{1,k} = h_{1,k}\left(W_1, \boldsymbol{V}_2^{k-1}\right) \tag{19}$$
$$h_{2,k} : \mathcal{W}_2 \times \mathcal{V}_1^{k-1} \to \mathcal{V}_{2,k}, \quad V_{2,k} = h_{2,k}\left(W_2, \boldsymbol{V}_1^{k-1}\right) \tag{20}$$

and limit the conference rates with the bounds

$$\sum_{k=1}^{K} \log(|\mathcal{V}_{1,k}|) \leq NC_{12} \tag{21}$$
$$\sum_{k=1}^{K} \log(|\mathcal{V}_{2,k}|) \leq NC_{21} \tag{22}$$

where $|\mathcal{V}_{t,k}|$ is the cardinality of $\mathcal{V}_{t,k}$.

The encoding function f_t of user t maps W_t and what was learned from the conference into a codeword $\boldsymbol{x}_t$. A code has two sets of K functions (19)–(20), two encoding functions

$$f_1 : \mathcal{W}_1 \times \mathcal{V}_2^K \to \mathcal{X}_1^N \tag{23}$$
$$f_2 : \mathcal{W}_2 \times \mathcal{V}_1^K \to \mathcal{X}_2^N \tag{24}$$

that generate codewords

$$\boldsymbol{X}_1 = f_1\left(W_1, \boldsymbol{V}_2^K\right) \tag{25}$$
$$\boldsymbol{X}_2 = f_2\left(W_2, \boldsymbol{V}_1^K\right) \tag{26}$$

and two decoding functions

$$(\hat{W}_1(t), \hat{W}_2(t)) = g_t(\boldsymbol{Y}_t), \qquad t = 1, 2 \tag{27}$$

such that the error probability is

$$P_e = \sum_{(w_1,w_2)\in\mathcal{W}_1\times\mathcal{W}_2} \frac{1}{M_1M_2} P\Bigg[\{g_1(\boldsymbol{Y}_1) \neq (w_1, w_2)\} \cup \{g_2(\boldsymbol{Y}_2) \neq (w_1, w_2)\}|(w_1, w_2) \text{ sent}\Bigg]. \tag{28}$$

A rate pair (R_1, R_2) is achievable if, for any $\epsilon > 0$, there exists an (M_1, M_2, N, K, P_e) code such that

$$M_t \geq 2^{NR_t}, \qquad t = 1, 2, \text{ and } P_e \leq \epsilon. \tag{29}$$

The capacity region of the compound MAC with conferencing encoders is the closure of the set of all achievable rate pairs (R_1, R_2).

Theorem 2: The capacity region of the compound MAC with conferencing links with capacities C_{12} and C_{21} is

$$\begin{aligned}\mathcal{C}(C_{12}, C_{21}) = \bigcup\Big\{&(R_1, R_2) : R_1 \geq 0, R_2 \geq 0,\\ &R_1 \leq \min\{I(X_1; Y_1|X_2, U), I(X_1; Y_2|X_2, U)\} + C_{12},\\ &R_2 \leq \min\{I(X_2; Y_1|X_1, U), I(X_2; Y_2|X_1, U)\} + C_{21},\\ &R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1|U), I(X_1, X_2; Y_2|U)\}\\ &\qquad + C_{12} + C_{21},\\ &R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1), I(X_1, X_2; Y_2)\}\Big\}\end{aligned} \tag{30}$$

where the union is over all joint distributions that factor as

$$p(u,x_1,x_2,y_t)=p(u)p(x_1|u)p(x_2|u)p(y_t|x_1,x_2),\qquad t=1,2. \quad (31)$$

Proof: For the converse, the same reasoning as in the proof of the converse in Theorem 1 applies.

For achievability, we use the strategy proposed by Willems in [8, Sec. IV]. Specifically, we partition the set $\{1,\ldots,M_1\}$ into $2^{NR_{12}}$ cells that we label as $s_1 \in \{1,\ldots,2^{NR_{12}}\}$. Each cell has $2^{N(R_1-R_{12})}$ elements that we label as $t_1\in\{1,\ldots,2^{N(R_1-R_{12})}\}$. Define $c_1(w_1)=s_1$, if w_1 is in cell s_1. The same type of partitioning is done for $\{1,2,\ldots M_2\}$. Define $R_{12}=\min\{R_1,C_{12}\}\le C_{12}$, $R_{21}=\min\{R_2,C_{21}\}\le C_{21}$, and

$$w_0'=(w_{01}',w_{02}')=(c_1(w_1),c_2(w_2)). \quad (32)$$

We refer to W_0' as a *common* message. Note that t_1 and t_2 are unknown to encoders 2 and 1, respectively.

For a single receiver, the MAC after the conference thus reduces to a MAC with common and private messages at the encoders [8]. The achievability of $\mathcal{R}_{\mathrm{MAC},t}$ in (7) then guarantees that the rates for the MAC with conferencing in [8, Sec. II] are achievable. Similarly, the compound MAC with conferencing after the conference is identical to the compound MAC with common information with W_0, W_1, W_2 replaced by (s_1,s_2), t_1, t_2, respectively. The common rate is thus $R_{12}+R_{21}$ and the private rates are R_1-R_{12} and R_2-R_{21}. It follows from Theorem 1 that (R_1,R_2) is achievable for the compound MAC with conferencing if $(R_{12}+R_{21},R_1-R_{12},R_2-R_{21})$ belongs to $\mathcal{C}_{\mathrm{CMAC}}$ in (12). □

We have the following relationship between the region (30) and the CMAC capacity region (12):

$$c(R_1,R_2)=c_{\mathrm{CMAC}}(R_{12}+R_{21},R_1-R_{12},R_2-R_{21}) \quad (33)$$

where $R_{12}=\min\{C_{12},R_1\}$, $R_{21}=\min\{C_{21},R_2\}$, and the functions $c_{\mathrm{CMAC}}(\cdot)$ and $c(\cdot)$ are defined in (1).

A. Discussion

Observe that $\mathcal{C}(C_{12}=0,C_{21}=0)$ is the capacity region of the two-sender, two-receiver channel with noncooperating encoders established by Ahlswede [16]. The rates (30) quantify the improvement due to transmitter cooperation over the conference links. We can further apply Theorem 2 to a Gaussian channel in the standard form [2], [21]

$$Y_1=X_1+h_{21}X_2+Z_1 \quad (34)$$
$$Y_2=h_{12}X_1+X_2+Z_2 \quad (35)$$

where the Z_t are independent, zero-mean, unit-variance Gaussian random variables and h_{12} and h_{21} are real numbers. We further add the power constraints

$$\frac{1}{N}\sum_{i=1}^{N}E\left[X_{ti}^2\right]\le P_t,\qquad t=1,2. \quad (36)$$

The power expended for the conference is not considered. We have the following result.

Corollary 1: The capacity region of the Gaussian compound MAC with conferencing encoders is

$$\mathcal{C}_G(C_{12},C_{21})=\bigcup\Big\{(R_1,R_2): R_1\ge 0, R_2\ge 0,$$
$$R_1\le \min\left\{C(\bar{a}P_1),C\left(h_{12}^2\bar{a}P_1\right)\right\}+C_{12}, \quad (37)$$
$$R_2\le \min\left\{C(\bar{b}P_2),C\left(h_{21}^2\bar{b}P_2\right)\right\}+C_{21}, \quad (38)$$
$$R_1+R_2\le \min\left\{C\left(\bar{a}P_1+h_{21}^2\bar{b}P_2\right),C\left(h_{12}^2\bar{a}P_1+\bar{b}P_2\right)\right\}+C_{12}+C_{21}, \quad (39)$$
$$R_1+R_2\le \min\Big\{C\left(P_1+h_{21}^2P_2+2sh_{21}\sqrt{aP_1bP_2}\right),\; C\left(h_{12}^2P_1+P_2+2sh_{12}\sqrt{aP_1bP_2}\right)\Big\}\Big\} \quad (40)$$

where the union is over all a, b, s, where $0\le a\le 1$, $0\le b\le 1$, $\bar{a}=1-a$, $\bar{b}=1-b$, $s=\pm 1$, and

$$C(x)=\frac{1}{2}\log(1+x).$$

Proof: All the mutual information expressions in (30) can be written as

$$I(X_{\mathcal{S}};Y_t|X_{\mathcal{S}^c},V)=H(Y_t|X_{\mathcal{S}^c},V)-H(Z_t) \quad (41)$$

for some V and set $\mathcal{S}$, where $\mathcal{S}^c$ denotes the complement of $\mathcal{S}$ in $\{1,2\}$. The maximum entropy theorem [20, Theorem 9.6.5] thus implies that Gaussian inputs maximize (41). Evaluating (30) for Gaussian inputs yields (37)–(40). □

Fig. 4 shows the capacity region for a symmetric channel where $C_{12}=C_{21}=c=0.5$, $P_1=P_2=P=10$, $h_{12}=h_{21}=h=0.89$. Due to the symmetry, we can choose $a=b$. Fig. 4 shows that cooperation gives substantial gains over no cooperation $(c=0)$. Note that the bounds (37)–(39) are loosest for $a=0$. As a increases, these bounds become more restrictive, but the bound (40) becomes looser. The sum rate is maximized when a is chosen such that (39) and (40) are the same. The capacity region is the union of all the pentagons obtained for different values of a.

IV. The Capacity Region of the Strong Interference Channel With Common Information

Suppose we relax the constraints of Section II, where both receivers decode both private messages. Instead, suppose decoder t is interested in only the common message and the private message of encoder t (see Fig. 5). We refer to this channel as an ICCI. We determine the capacity region of ICCIs if

$$I(X_1;Y_1|X_2,U)\le I(X_1;Y_2|X_2,U) \quad (42)$$
$$I(X_2;Y_2|X_1,U)\le I(X_2;Y_1|X_1,U) \quad (43)$$

for all joint distributions $p(u,x_1,x_2,y_1,y_2)$ that factor as $p(u)p(x_1|u)p(x_2|u)p(y_1,y_2|x_1,x_2)$. We further show that this class of interference channels is same as those determined by (48) and (49) below with independent X_1 and X_2.

The encoding is done as in Section II. However, each decoder t now estimates the common message W_0 and the private message W_t based on the received N-sequence $\boldsymbol{Y}_t$ as

$$(\hat{W}_0(1),\hat{W}_1)=g_1(\boldsymbol{Y}_1) \quad (44)$$
$$(\hat{W}_0(2),\hat{W}_2)=g_2(\boldsymbol{Y}_2). \quad (45)$$

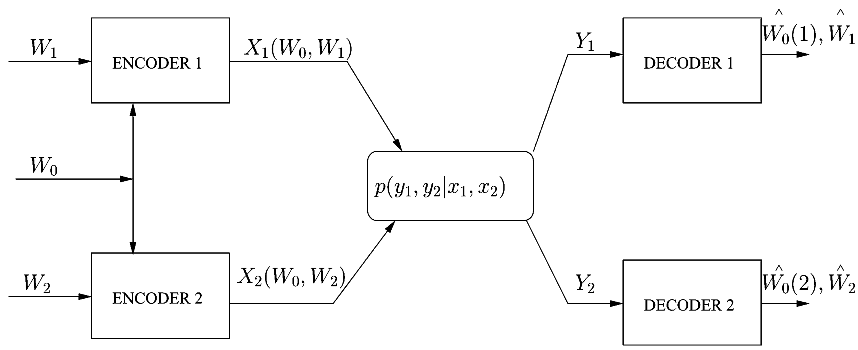

Fig. 5. Interference channel with common information.

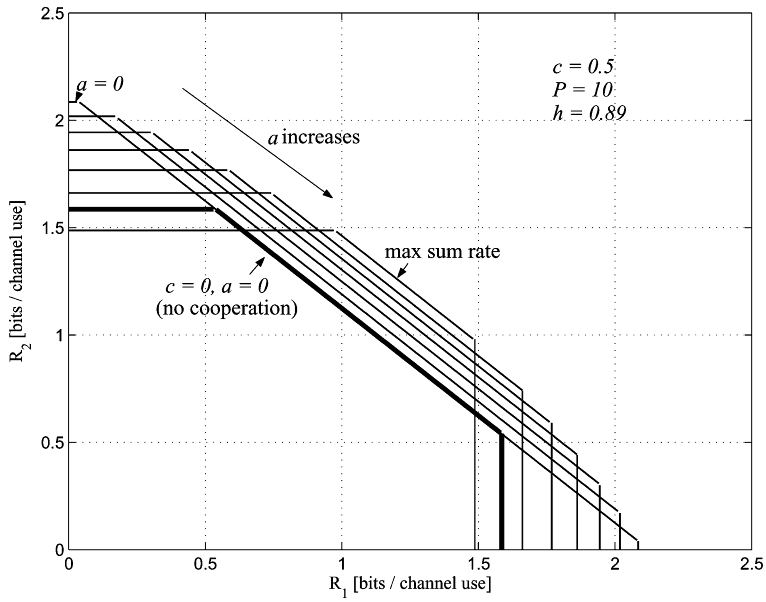

Fig. 4. The Gaussian compound MAC with conferencing encoders capacity region with parameters $h_{12} = h_{21} = 0.89$, $P_1 = P_2 = P = 10$, and $C_{12} = C_{21} = c = 0.5$.

An (M_0, M_1, M_2, N, P_e) code for the channel has two encoding functions f_1, f_2, two decoding functions g_1, g_2, and an error probability

$$P_e \triangleq \max\{P_{e,1}, P_{e,2}\} \tag{46}$$

where

$$P_{e,t} = \sum_{(w_0,w_1,w_2)} \frac{1}{M_0 M_1 M_2} P\Big[g_t(\boldsymbol{Y}_t) \neq (w_0, w_t) \,|(w_0, w_1, w_2) \text{ sent}\Big]. \tag{47}$$

We remark that we could alternatively define P_e by using a union of events as in (5). A rate triple (R_0, R_1, R_2) is achievable if, for any $\epsilon > 0$, there is an (M_0, M_1, M_2, N, P_e) code such that

$$M_t \geq 2^{NR_t}, \qquad t = 0, 1, 2, \text{ and } P_e \leq \epsilon.$$

The capacity region of the ICCI is the closure of the set of all achievable rate triples (R_0, R_1, R_2). We have the following result.

Theorem 3: An ICCI satisfying the strong interference conditions [13]

$$I(X_1; Y_1|X_2) \leq I(X_1; Y_2|X_2) \tag{48}$$
$$I(X_2; Y_2|X_1) \leq I(X_2; Y_1|X_1) \tag{49}$$

for all joint distributions $p(x_1, x_2, y_1, y_2)$ that factor as $p(x_1)p(x_2)p(y_1, y_2|x_1, x_2)$ has the capacity region

$$\mathcal{C}_{\text{ICCI}} = \bigcup\{(R_0, R_1, R_2) : R_0 \geq 0, R_1 \geq 0, R_2 \geq 0\}$$
$$R_1 \leq I(X_1; Y_1|X_2, U) \tag{50}$$
$$R_2 \leq I(X_2; Y_2|X_1, U) \tag{51}$$
$$R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1|U), I(X_1, X_2; Y_2|U)\} \tag{52}$$
$$R_0 + R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1), I(X_1, X_2; Y_2)\} \tag{53}$$

where the union is over all joint distributions that factor as

$$p(u, x_1, x_2, y_1, y_2) = p(u)p(x_1|u)p(x_2|u)p(y_1, y_2|x_1, x_2). \tag{54}$$

When the constraints (48) and (49) are satisfied, we refer to the channel as an ICCI with strong interference. Furthemore, we have

$$c_{\text{ICCI}}(R_0, R_1, R_2) = c_{\text{CMAC}}(R_0, R_1, R_2).$$

Proof: From Theorem 1, the rates (12) are achievable if both private messages are decoded at the receivers. These rates are clearly also achieved if only a single private message must be decoded, and are hence achievable in the ICCI. The converse is more involved and is given in Section IV-A.

A. Converse: Strong Interference Conditions

We will need the data processing inequality in the following form.

Lemma 1: For a Markov chain $W \to (U, X) \to Y$, we have

$$I(W;Y|U) \le I(X;Y|U). \tag{55}$$

Proof: We have

$$\begin{aligned} H(Y|U,X) &=^{(a)} H(Y|W,U,X) \\ &\le^{(b)} H(Y|W,U) \end{aligned} \tag{56}$$

where (a) holds because of the Markov property and (b) since conditioning cannot increase entropy. Subtracting both sides from $H(Y|U)$ gives the desired result. □

To obtain per-letter conditions (48)–(49) in Theorem 3 we will need the following Lemmas.

Lemma 2: If

$$I(X_1;Y_1|X_2,U) \le I(X_1;Y_2|X_2,U) \tag{57}$$

for all probability distributions on $\mathcal{U} \times \mathcal{X}_1 \times \mathcal{X}_2$ such that $p(u,x_1,x_2) = p(u)p(x_1|u)p(x_2|u)$, then

$$I(\boldsymbol{X}_1;\boldsymbol{Y}_1|\boldsymbol{X}_2,\boldsymbol{U}) \le I(\boldsymbol{X}_1;\boldsymbol{Y}_2|\boldsymbol{X}_2,\boldsymbol{U}). \tag{58}$$

We note that by symmetry, it follows from Lemma 2 that

$$I(X_2;Y_2|X_1,U) \le I(X_2;Y_1|X_1,U) \tag{59}$$

implies $I(\boldsymbol{X}_2;\boldsymbol{Y}_2|\boldsymbol{X}_1,\boldsymbol{U}) \le I(\boldsymbol{X}_2;\boldsymbol{Y}_1|\boldsymbol{X}_1,\boldsymbol{U})$.

Proof: The proof follows the same approach as that in the proof of [22, Proposition 1] and that in [13, Lemma]. Furthermore, the proof is very similar to the proof of Lemma 5 presented in the Appendix and is therefore omitted. □

We will also need the following Lemma.

Lemma 3: The class of interference channels $p(y_1,y_2|x_1,x_2)$ for which (57) and (59) are valid for all distributions $p(u,x_1,x_2,y_1,y_2)$ that factor as in (54) is the same as the class of $p(y_1,y_2|x_1,x_2)$ for which (48) and (49) are valid for all product input distributions $p(x_1,x_2) = p(x_1)p(x_2)$.

Proof: We use the following result from [13, Lemma]: If $I(X_1;Y_1|X_2) \le I(X_1;Y_2|X_2)$ for all product probability distributions on $\mathcal{X}_1 \times \mathcal{X}_2$, then $I(X_1;Y_1|X_2,U) \le I(X_1;Y_2|X_2,U)$ where $U \to (X_1,X_2) \to (Y_1,Y_2)$ and $X_1 \to U \to X_2$. The strong interference conditions (48) and (49) thus imply the conditions (57) and (59).

To prove the converse, we observe that since (57) and (59) are satisfied for all distributions of the form (54), the conditions (57) and (59) must also hold for U being independent of X_1 and X_2. For such distributions $p(u,x_1,x_2)$, conditions (57) and (59) reduce to (48) and (49). □

Lemmas 2 and 3 together imply the following statement.

Lemma 4: If

$$I(X_1;Y_1|X_2) \le I(X_1;Y_2|X_2) \tag{60}$$

for all product input distributions $p(x_1,x_2) = p(x_1)p(x_2)$, then

$$I(\boldsymbol{X}_1;\boldsymbol{Y}_1|\boldsymbol{X}_2,\boldsymbol{U}) \le I(\boldsymbol{X}_1;\boldsymbol{Y}_2|\boldsymbol{X}_2,\boldsymbol{U}). \tag{61}$$

We next prove the converse in Theorem 3.

Proof: Consider an (M_0, M_1, M_2, N, P_e) code for the ICCI. Applying Fano's inequality gives

$$\begin{aligned} H(W_0,W_1|\boldsymbol{Y}_1) &\le P_{e,1}\log(M_0M_1-1)+h(P_{e,1}) \triangleq N\delta_{1,N} \\ H(W_0,W_2|\boldsymbol{Y}_2) &\le P_{e,2}\log(M_0M_2-1)+h(P_{e,2}) \triangleq N\delta_{2,N}. \end{aligned}$$

where $\delta_{t,N} \to 0$ as $P_{e,t} \to 0$ (or as $P_e \to 0$). It follows that

$$H(W_0,W_1|\boldsymbol{Y}_1) = H(W_0|\boldsymbol{Y}_1) + H(W_1|\boldsymbol{Y}_1,W_0) \le N\delta_{1,N} \tag{62}$$

$$H(W_0,W_2|\boldsymbol{Y}_2) = H(W_0|\boldsymbol{Y}_2) + H(W_2|\boldsymbol{Y}_2,W_0) \le N\delta_{2,N}. \tag{63}$$

Since conditioning cannot increase entropy, from (63) it follows that

$$H(W_2|\boldsymbol{Y}_2,W_0,W_1) \le H(W_2|\boldsymbol{Y}_2,W_0) \le N\delta_{2,N}. \tag{64}$$

From the problem definition, we have the following Markov chains for the ICCI:

$$W_1 \to (\boldsymbol{X}_1, W_0) \to \boldsymbol{Y}_2 \tag{65}$$

$$W_2 \to (\boldsymbol{X}_2, W_0, W_1) \to \boldsymbol{Y}_2 \tag{66}$$

$$(W_0, W_t) \to (\boldsymbol{X}_t, W_0) \to \boldsymbol{Y}_t, \qquad t = 1,2. \tag{67}$$

Applying Lemma 1 to the Markov Chains (66)–(67) yields

$$I(W_2;\boldsymbol{Y}_2|W_0,W_1) \le I(\boldsymbol{X}_2;\boldsymbol{Y}_2|W_0,W_1) \tag{68}$$

$$I(W_t;\boldsymbol{Y}_t|W_0) \le I(\boldsymbol{X}_t;\boldsymbol{Y}_t|W_0). \tag{69}$$

Using (67) with $t = 1$ yields

$$I(W_0,W_1;\boldsymbol{Y}_1) \le I(W_0,\boldsymbol{X}_1;\boldsymbol{Y}_1). \tag{70}$$

We first consider the bound (53) at decoder 1. Inequalities (62) and (64) imply that for reliable communication we require

$$\begin{aligned} &N(R_0+R_1+R_2) \\ &\quad \le^{(a)} I(W_0,W_1;\boldsymbol{Y}_1) + I(W_2;\boldsymbol{Y}_2|W_0,W_1) \\ &\quad \le^{(b)} I(W_0,\boldsymbol{X}_1;\boldsymbol{Y}_1) + I(\boldsymbol{X}_2;\boldsymbol{Y}_2|W_0,W_1) \\ &\quad \le^{(c)} I(W_0,\boldsymbol{X}_1;\boldsymbol{Y}_1) + I(\boldsymbol{X}_2;\boldsymbol{Y}_2|W_0,W_1,\boldsymbol{X}_1(W_0,W_1)) \\ &\quad \le^{(d)} I(W_0,\boldsymbol{X}_1;\boldsymbol{Y}_1) + I(\boldsymbol{X}_2;\boldsymbol{Y}_2|W_0,\boldsymbol{X}_1) \end{aligned} \tag{71}$$

where (a) follows from the independence of W_0, W_1, W_2; (b) by (68) and (70); (c) by (3); and (d) by (65). If

$$I(\boldsymbol{X}_2;\boldsymbol{Y}_2|W_0,\boldsymbol{X}_1) \le I(\boldsymbol{X}_2;\boldsymbol{Y}_1|W_0,\boldsymbol{X}_1) \tag{72}$$

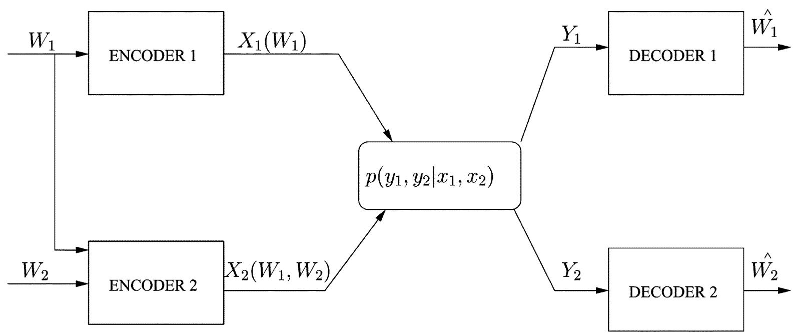

Fig. 6. Interference channel with unidirectional cooperation.

then it follows by (71) that

$$\begin{aligned} N(R_0+R_1+R_2) &\le I(W_0, \boldsymbol{X}_1; \boldsymbol{Y}_1) + I(\boldsymbol{X}_2; \boldsymbol{Y}_1|\boldsymbol{X}_1, W_0) \\ &= I(W_0, \boldsymbol{X}_1, \boldsymbol{X}_2; \boldsymbol{Y}_1) \\ &= I(\boldsymbol{X}_1, \boldsymbol{X}_2; \boldsymbol{Y}_1) \\ &\le \sum_{n=1}^{N} I(X_{1n}, X_{2n}; Y_{1n}). \end{aligned} \tag{73}$$

We define

$$U_n = W_0, \qquad n = 1, \ldots, N \tag{74}$$

so that condition (72) becomes

$$I(\boldsymbol{X}_2; \boldsymbol{Y}_2|\boldsymbol{X}_1, \boldsymbol{U}) \le I(\boldsymbol{X}_2; \boldsymbol{Y}_1|\boldsymbol{X}_1, \boldsymbol{U}). \tag{75}$$

We next prove the bound (52). Fano's inequality implies

$$\begin{aligned} &N(R_1+R_2) \\ &\quad\le I(W_1; \boldsymbol{Y}_1) + I(W_2; \boldsymbol{Y}_2) \\ &\quad\le^{(a)} I(W_1; \boldsymbol{Y}_1|W_0) + I(W_2; \boldsymbol{Y}_2|W_0, W_1) \\ &\quad\le^{(b)} I(\boldsymbol{X}_1; \boldsymbol{Y}_1|W_0) + I(\boldsymbol{X}_2; \boldsymbol{Y}_2|W_0, W_1) \\ &\quad=^{(c)} I(\boldsymbol{X}_1; \boldsymbol{Y}_1|W_0) + I(\boldsymbol{X}_2; \boldsymbol{Y}_2|W_0, W_1, \boldsymbol{X}_1(W_0, W_1)) \\ &\quad=^{(d)} I(\boldsymbol{X}_1; \boldsymbol{Y}_1|\boldsymbol{U}) + I(\boldsymbol{X}_2; \boldsymbol{Y}_2|\boldsymbol{X}_1, \boldsymbol{U}) \end{aligned} \tag{76}$$

where again (a) follows from the independence of W_0, W_1, W_2; (b) by (68) and (69); (c) by (3); (d) by (65) and (74). Again, if (72) holds, then (76) becomes

$$\begin{aligned} N(R_1+R_2) &\le I(\boldsymbol{X}_1; \boldsymbol{Y}_1|\boldsymbol{U}) + I(\boldsymbol{X}_2; \boldsymbol{Y}_1|\boldsymbol{X}_1, \boldsymbol{U}) \\ &= I(\boldsymbol{X}_1, \boldsymbol{X}_2; \boldsymbol{Y}_1|\boldsymbol{U}) \\ &\le \sum_{n=1}^{N} I(X_{1n}, X_{2n}; Y_{1n}|U_n). \end{aligned} \tag{77}$$

The same approach can be used to show that the bounds (52) and (53) are satisfied at decoder 2 under a condition similar to (75), namely

$$I(\boldsymbol{X}_1; \boldsymbol{Y}_1|\boldsymbol{X}_2, \boldsymbol{U}) \le I(\boldsymbol{X}_1; \boldsymbol{Y}_2|\boldsymbol{X}_2, \boldsymbol{U}). \tag{78}$$

From Lemma 4, it follows that the per-letter conditions (48) and (49) imply (75) and (78). The bounds (50) and (51) follow by standard methods as in [10, Sec. III.4]. Finally, we observe from (73) and (77) that $(R_0, R_1, R_2) \in co(\mathcal{C}_{\mathrm{ICCI}})$. Since the region $\mathcal{C}_{\mathrm{ICCI}}$ is already convex, we have $(R_0, R_1, R_2) \in \mathcal{C}_{\mathrm{ICCI}}$. □

B. Gaussian Channel

Theorem 4: For the Gaussian ICCI (34)–(35) with the power constraints (36) that satisfy $h_{12}^2 \ge 1$, $h_{21}^2 \ge 1$, the capacity region is

$$\begin{aligned} \mathcal{C}_G = \bigcup \Big\{ &(R_0, R_1, R_2) : R_0 \ge 0, R_1 \ge 0, R_2 \ge 0, \\ &R_1 \le C(\bar{a}P_1), \\ &R_2 \le C(\bar{b}P_2), \\ &R_1 + R_2 \le \min\left\{C\left(\bar{a}P_1 + h_{21}^2\bar{b}P_2\right), C\left(h_{12}^2\bar{a}P_1 + \bar{b}P_2\right)\right\}, \\ &R_0 + R_1 + R_2 \le \min\Big\{C\left(P_1 + h_{21}^2P_2 + 2sh_{21}\sqrt{aP_1bP_2}\right), \\ &C\left(h_{12}^2P_1 + P_2 + 2sh_{12}\sqrt{aP_1bP_2}\right)\Big\}\Big\} \end{aligned} \tag{79}$$

where the union is over all a, b, s, where $0 \le a \le 1, 0 \le b \le 1$, $\bar{a} = 1 - a$, $\bar{b} = 1 - b$, and $s = \pm 1$.

Proof: We can use similar reasoning as in [12] to show that both users can decode all three messages. Suppose (R_0, R_1, R_2) is inside the capacity region and the transmitters use codes that reliably achieve these rates. Receiver 1 can then generate x_1 since it must decode both w_0 and w_1. Similarly, receiver 2 can generate x_2, and receivers 1 and 2 can generate the respective

$$Y_1' = h_{12}X_1 + X_2 + Z_1/h_{21} \tag{80}$$

$$Y_2' = X_1 + h_{21}X_2 + Z_2/h_{12}. \tag{81}$$

But observe that (80) and (81) are less noisy versions of (35) and (34), respectively. Hence both receivers can decode all three messages and the channel becomes a compound MAC. From the maximum-entropy theorem [20, Theorem 9.6.5], the compound MAC region (12) is largest for Gaussian inputs. Evaluating (12) for Gaussian inputs and $h_{12}^2 \ge 1$, $h_{21}^2 \ge 1$ yields (79). □

V. The Capacity Region of the Strong Interference Channel With Unidirectional Cooperation

We now consider an interference channel where the message sent at one encoder is available to the other encoder, but not *vice versa*. Achievable rates for this channel model have been presented in [23]. The channel was referred to as a cognitive radio channel because of its relation to cognitive radio applications. Furthermore, for the case of weak interference, i.e., $h_{21} < 1$, the capacity region was determined in [14] and [15]. The communication system is shown in Fig. 6, and we describe the encoders, decoders, and error probability next.

Each encoder t wishes to send an independent message $W_t \in \{1,\ldots,M_t\}$ to receiver t in N channel uses. Message W_1 is also known at encoder 2, thus allowing for unidirectional cooperation. The channel is memoryless and time-invariant as given by (2). An (M_1,M_2,N,P_e) code for the channel has two encoding functions generating codewords

$$\boldsymbol{X}_1 = f_1(W_1) \quad (82)$$
$$\boldsymbol{X}_2 = f_2(W_1,W_2) \quad (83)$$

two decoding functions

$$\hat{W}_t = g_t(\boldsymbol{Y}_t) \quad t=1,2 \quad (84)$$

and an error probability

$$P_e = \max\{P_{e,1},P_{e,2}\} \quad (85)$$

where, for $t = 1, 2$, we have

$$P_{e,t} = \sum_{(w_1,w_2)} \frac{1}{M_1M_2} P\left[g_t(\boldsymbol{Y}_t) \neq (w_t) | (w_1,w_2) \text{ sent}\right]. \quad (86)$$

A rate pair (R_1,R_2) is achievable if, for any $\epsilon > 0$, there is an (M_1,M_2,N,P_e) code such that

$$M_t \geq 2^{NR_t}, \qquad t=1,2, \text{ and } P_e \leq \epsilon.$$

The capacity region of the ICUC is the closure of the set of all achievable rate pairs (R_1,R_2).

Theorem 5: An ICUC that satisfies the strong interference conditions

$$I(X_2;Y_2|X_1) \leq I(X_2;Y_1|X_1) \quad (87)$$
$$I(X_1,X_2;Y_1) \leq I(X_1,X_2;Y_2) \quad (88)$$

for all input distributions $p(x_1,x_2)$ has the capacity region

$$\mathcal{C}_{\text{ICUC}} = \bigcup \left\{ \begin{aligned} &(R_1,R_2): R_1 \geq 0, R_2 \geq 0, \\ &R_2 \leq I(X_2;Y_2|X_1) \quad (89) \\ &R_1+R_2 \leq I(X_1,X_2;Y_1) \end{aligned} \right\} \quad (90)$$

where the union is over all input distributions $p(x_1,x_2)$.

We prove Theorem 5 in Sections V-A and -B. In Section V-A, we indicate how the achievability of $\mathcal{C}_{\text{ICUC}}$ follows from the compound MAC with common information. In Section V-B, we prove the converse and determine the strong interference conditions. From Theorems 1 and 5, we further have

$$c_{\text{ICUC}}(R_1,R_2) = c_{\text{CMAC}}(R_1,0,R_2). \quad (91)$$

A. Achievability

We apply the same reasoning as in Section IV. The rates (12) of Theorem 1 are achievable when the decoders decode the common message and both private messages. Encoder 2 knows W_1, so we can view R_1 as the common rate. For the same reason, W_1 in the corresponding CMAC has zero rate for the private message, also reflected in (91). We choose $U = X_1$ and the region (12) becomes

$$\mathcal{C}_{\text{CMAC}} = \bigcup \left\{ \begin{aligned} &(R_1,R_2): R_1 \geq 0, R_2 \geq 0, \\ &R_2 \leq \min_t I(X_2;Y_t|X_1) \\ &R_1+R_2 \leq \min_t I(X_1,X_2;Y_t) \end{aligned} \right\} \quad (92)$$

where the union is over all $p(x_1,x_2,y_1,y_2)$. When conditions (87)–(88) are satisfied, the region (92) reduces to the region (89)–(90) in Theorem 5.

B. Converse: Strong Interference Conditions

To prove the converse, we will need the following lemma.

Lemma 5: If

$$I(X_2;Y_2|X_1) \leq I(X_2;Y_1|X_1) \quad (93)$$

holds for all joint distributions $p(x_1,x_2)$, then

$$I(\boldsymbol{X}_2;\boldsymbol{Y}_2|\boldsymbol{X}_1,\boldsymbol{U}) \leq I(\boldsymbol{X}_2;\boldsymbol{Y}_1|\boldsymbol{X}_1,\boldsymbol{U}). \quad (94)$$

By symmetry, we have that $I(X_1;Y_1|X_2) \leq I(X_1;Y_2|X_2)$ implies $I(\boldsymbol{X}_1;\boldsymbol{Y}_1|\boldsymbol{X}_2,\boldsymbol{U}) \leq I(\boldsymbol{X}_1;\boldsymbol{Y}_2|\boldsymbol{X}_2,\boldsymbol{U})$.

Proof: See the Appendix.

Lemma 5 is similar to a Lemma by Costa and El Gamal [13], as further explained in the Appendix. We point out that the only difference between Lemmas 4 and 5 is the constraint on the probability distributions. The direct proof of Lemma 4 follows exactly the same steps as the proof of Lemma 5 presented in the Appendix.

We next prove the converse in Theorem 5.

Proof: Consider an (M_1,M_2,N,P_e) code for the ICUC. Applying Fano's inequality gives

$$H(W_1|\boldsymbol{Y}_1) \leq P_{e,1}\log(M_1-1) + h(P_{e,1}) \triangleq N\delta_{1,N} \quad (95)$$
$$H(W_2|\boldsymbol{Y}_2) \leq P_{e,2}\log(M_2-1) + h(P_{e,2}) \triangleq N\delta_{2,N} \quad (96)$$

where $\delta_{t,N} \to 0$ as $P_{e,t} \to 0$ (or as $P_e \to 0$). We now derive the bound (90) for receiver $t=1$.

The inequalities (95) and (96) imply that for reliable communication we require

$$\begin{aligned} N(R_1+R_2) &\leq I(W_1;\boldsymbol{Y}_1) + I(W_2;\boldsymbol{Y}_2) \\ &\leq^{(a)} I(W_1;\boldsymbol{Y}_1) + I(W_2;\boldsymbol{Y}_2|W_1) \\ &=^{(b)} I(W_1;\boldsymbol{Y}_1) + I(W_2,\boldsymbol{X}_2;\boldsymbol{Y}_2|\boldsymbol{X}_1,W_1) \\ &\leq^{(c)} I(W_1,\boldsymbol{X}_1;\boldsymbol{Y}_1) + I(\boldsymbol{X}_2;\boldsymbol{Y}_2|\boldsymbol{X}_1,W_1) \end{aligned} \quad (97)$$

where (a) follows by the independence of W_1 and W_2; (b) by (82) and (83); (c) by the Markov chain $(W_1,W_2) \to (\boldsymbol{X}_1,\boldsymbol{X}_2) \to \boldsymbol{Y}_2$. But, from Lemma 5 we have

$$I(\boldsymbol{X}_2;\boldsymbol{Y}_2|\boldsymbol{X}_1,W_1) \leq I(\boldsymbol{X}_2;\boldsymbol{Y}_1|\boldsymbol{X}_1,W_1) \quad (98)$$

and it follows from (97) that

$$\begin{aligned} N(R_1+R_2) &\le I(W_1,\boldsymbol{X}_1;\boldsymbol{Y}_1)+I(\boldsymbol{X}_2;\boldsymbol{Y}_1|\boldsymbol{X}_1,W_1) \\ &= I(\boldsymbol{X}_1,\boldsymbol{X}_2;\boldsymbol{Y}_1) \\ &\le \sum_{n=1}^{N} I(X_{1n},X_{2n};Y_{1n}). \end{aligned} \tag{99}$$

The bound (89) follows by standard methods

$$\begin{aligned} NR_2 &\le I(W_2;\boldsymbol{Y}_2) \\ &\le I(W_2;\boldsymbol{Y}_2|W_1) \\ &= I(W_2,\boldsymbol{X}_2;\boldsymbol{Y}_2|\boldsymbol{X}_1,W_1) \\ &\le \sum_{n=1}^{N} I(X_{2n};Y_{2n}|X_{1n}) \end{aligned} \tag{100}$$

and now we convert (100) to the single-letter bound (89) as in [10, Sec. III.4]. This concludes the proof of Theorem 5. □

We remark that if (88) does not hold, showing that (89)–(90) is the capacity region would require proving an outer bound of the form $R_1+R_2 \le I(X_1,X_2;Y_2)$. Due to the asymmetry of the problem, the approach of (97)–(99) does not apply.

C. Gaussian Channel

Theorem 6: For the Gaussian ICUC (34)–(35) with the power constraints (36) that satisfy the Gaussian strong interference (GSI-UC) conditions

$$h_{21}^2 \ge 1 \tag{101}$$

$$|h_{12}\alpha+1| \ge |\alpha+h_{21}| \tag{102}$$

$$|h_{12}\alpha-1| \ge |\alpha-h_{21}| \tag{103}$$

where $\alpha=\sqrt{P_1/P_2}$, the capacity region is

$$\begin{aligned} \mathcal{C}_G = \bigcup_{|\rho|\le 1} \Big\{ &(R_1,R_2): R_1\ge 0, R_2\ge 0, \\ &R_2 \le C\left((1-\rho^2)P_2\right), \qquad (104) \\ &R_1+R_2 \le C\left(P_1+h_{21}^2P_2+2\rho h_{21}\sqrt{P_1P_2}\right)\Big\} \qquad (105) \end{aligned}$$

where ρ is the correlation coefficient for X_1, X_2.

Proof: For achievability, we observe from the maximum-entropy theorem [20, Theorem 9.6.5] that the compound MAC region (92) is largest with Gaussian inputs. Under conditions

$$h_{21}^2 \ge 1 \tag{106}$$

$$1+h_{12}^2\alpha^2+2h_{12}\alpha\rho \ge \alpha^2+h_{21}^2+2h_{21}\alpha\rho \tag{107}$$

the region (92) reduces to (104)–(105). For the inequality (107) to hold for all Gaussian inputs X_1, X_2, it must hold for all values of ρ. However, as (107) is linear in ρ, it is necessary and sufficient to enforce the constraint at $\rho=\pm 1$. This yields the conditions (102)–(103).

For the converse, following the same reasoning as in Theorem 4, we observe that under condition (101), decoder 1 can decode both messages (w_1,w_2). Hence, the sum rate is bounded by the cut-set bound [20, p. 445]

$$R_1+R_2 \le I(X_1,X_2;Y_1) \tag{108}$$

which is maximized for Gaussian inputs and evaluates to (105). The bound (104) follows by standard methods as in (100). □

The constraints (102) and (103) are perhaps difficult to interpret directly as strong interference conditions. To simplify these constraints, we first note a sign reversal symmetry: with the substitutions $h_{12}=-\tilde{h}_{12}$ and $h_{21}=-\tilde{h}_{21}$, we obtain, not surprisingly, the same strong interference conditions in the parameters $\tilde{h}_{12}$ and $\tilde{h}_{21}$. From this symmetry, we can conclude it is sufficient to consider only the "same-sign" case with $h_{21}>0$ and $h_{12}>0$ and the "opposite-sign" case with $h_{21}>0$ and $h_{12}<0$. This facilitates the following claims.

Theorem 7: Let $|h_{21}|>1$.

(a) For $h_{12}h_{21}>0$, the GSI-UC conditions (102) and (103) hold if and only if

$$|h_{12}| \ge \frac{|\alpha-1|}{\alpha}+\frac{|h_{21}|}{\alpha}. \tag{109}$$

(b) For $h_{12}h_{21}<0$, the GSI-UC conditions (102) and (103) hold if and only if

$$|h_{12}| \ge \frac{\alpha+1}{\alpha}+\frac{|h_{21}|}{\alpha}. \tag{110}$$

A proof of Theorem 7 appears in the Appendix; the key is that the GSI-UC conditions always require $|h_{12}|\alpha \ge 1$. Note that the theorem imposes the strict inequality $|h_{21}|>1$, only because $h_{21}=1$ admits the possibility that $h_{12}=h_{21}=1$ and in this case, the GSI-UC conditions (102) and (103) are satisfied for all α.

We observe from Theorem 7 that in all cases we have a strong interference condition in that $|h_{12}| \ge 1$. We also note that these conditions are more demanding, particularly on h_{12}, than those of (48)–(49), which in the Gaussian case reduce to $h_{21} \ge 1$, $h_{12} \ge 1$. In the limit of large α, the minimum h_{12} is a decreasing function of α. Still, there will always be some set of values h_{21}, h_{12} that satisfy the conditions of Theorem 7 as long as $\alpha>0$. Note that $\alpha=0$ corresponds to $P_1=0$ for which the channel reduces to the broadcast channel from encoder 2. As the channel is degraded, there can be no strong interference conditions.

Fig. 7 shows the capacity region for $P_1=P_2=P=10$ and $h_{21}=1.5$. Note that $P_1=P_2$ implies $\alpha=1$ so that condition (109) of Theorem 7 reduces to $h_{12}\ge h_{21}$. Alternatively, in the opposite case, the GSI-UC conditions are satisfied if $-h_{12} \ge 1+h_{21}$.

VI. Conclusion

We presented three channel models that incorporate partial transmitter cooperation. For two interference channels presented in Sections IV and V, we determined the capacity region

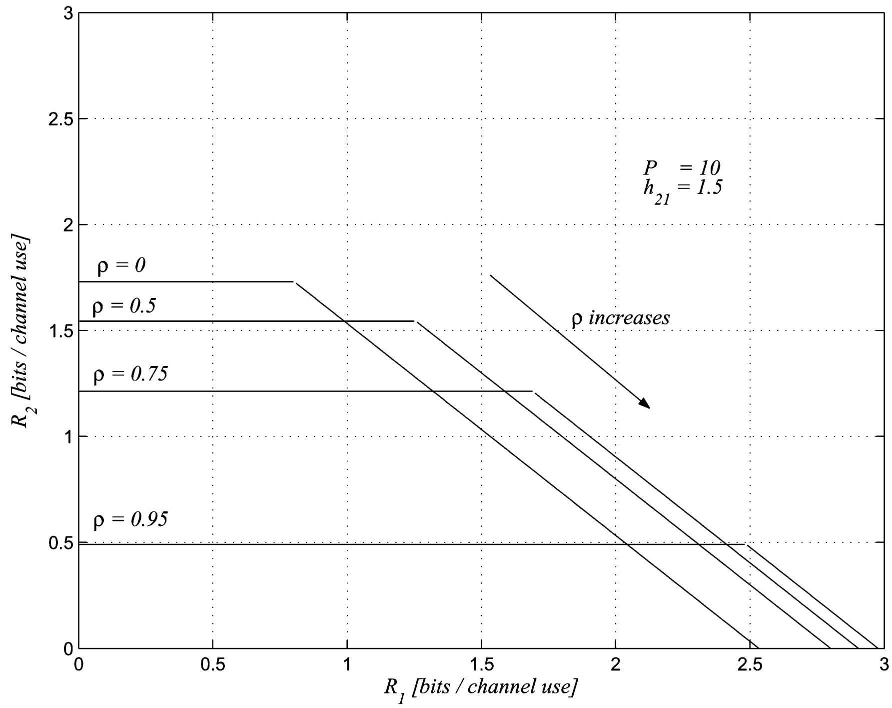

Fig. 7. Gaussian ICUC capacity region if $h_{12} \geq h_{21}$ or $h_{12} \leq -h_{21} - 1$.

under strong interference where the decoders can decode all messages with no rate penalty. For the channel with unidirectional cooperation, it is possible that weaker conditions exist for which the capacity region can be found. Determining the strong interference conditions for more general channel models such as interference channels with correlated sources is an open problem.

APPENDIX

Proof of Lemma 5: We will need a result similar to the ones in [22, Proposition 1] and [13, Lemma]. In fact, the only difference from the Proposition in [13] will be in the probability distributions for which the proposition holds.

Proposition 1: If

$$I(X_2;Y_2|X_1) \leq I(X_2;Y_1|X_1) \tag{111}$$

for all probability distributions on $\mathcal{X}_1 \times \mathcal{X}_2$ then

$$I(X_2;Y_2|X_1,V) \leq I(X_2;Y_1|X_1,V) \tag{112}$$

for all probability distributions on $\mathcal{V} \times \mathcal{X}_1 \times \mathcal{X}_2$.

Proof of Proposition 1: We write the right-hand side in (112) as

$$\begin{aligned} I(X_2;Y_1|X_1,V) &= \sum_v P(v) I(X_2;Y_1|X_1,V=v) \\ &\geq \sum_v P(v) I(X_2;Y_2|X_1,V=v) \\ &= I(X_2;Y_2|X_1,V) \end{aligned} \tag{113}$$

where the inequality follows by (111). □

We follow the approach as in [22, Proposition 1] and [13, Lemma] to obtain

$$\begin{aligned} &I(\boldsymbol{X}_2;\boldsymbol{Y}_1|\boldsymbol{X}_1,\boldsymbol{U}) - I(\boldsymbol{X}_2;\boldsymbol{Y}_2|\boldsymbol{X}_1,\boldsymbol{U}) \\ &\quad = I(X_{2N};Y_{1N}|\boldsymbol{X}_1,\boldsymbol{U},\boldsymbol{Y}_1^{N-1}) \\ &\qquad - I(X_{2N};Y_{2N}|\boldsymbol{X}_1,\boldsymbol{U},\boldsymbol{Y}_1^{N-1}) \\ &\qquad + I(\boldsymbol{X}_2^{N-1};\boldsymbol{Y}_1^{N-1}|\boldsymbol{X}_1,\boldsymbol{U},Y_{2N}) \\ &\qquad - I(\boldsymbol{X}_2^{N-1};\boldsymbol{Y}_2^{N-1}|\boldsymbol{X}_1,\boldsymbol{U},Y_{2N}). \end{aligned} \tag{114}$$

We write (114) in a slightly different form

$$\begin{aligned} &I(\boldsymbol{X}_2;\boldsymbol{Y}_1|\boldsymbol{X}_1,\boldsymbol{U}) - I(\boldsymbol{X}_2;\boldsymbol{Y}_2|\boldsymbol{X}_1,\boldsymbol{U}) \\ &\quad = I\left(X_{2N};Y_{1N}|X_{1N},\boldsymbol{U},\boldsymbol{X}_1^{N-1},\boldsymbol{Y}_1^{N-1}\right) \\ &\qquad - I\left(X_{2N};\boldsymbol{Y}_{2N}|X_{1N},\boldsymbol{U},\boldsymbol{X}_1^{N-1},Y_1^{N-1}\right) \\ &\qquad + I\left(\boldsymbol{X}_2^{N-1};\boldsymbol{Y}_1^{N-1}|\boldsymbol{X}_1^{N-1},\boldsymbol{U},X_{1N},Y_{2N}\right) \\ &\qquad - I\left(\boldsymbol{X}_2^{N-1};\boldsymbol{Y}_2^{N-1}|\boldsymbol{X}_1^{N-1},\boldsymbol{U},X_{1N},Y_{2N}\right). \end{aligned} \tag{115}$$

By letting $V = \left(\boldsymbol{U},\boldsymbol{X}_1^{N-1},\boldsymbol{Y}_1^{N-1}\right)$, we observe that Proposition 1 yields

$$\begin{aligned} &I\left(X_{2N};Y_{1N}|X_{1N},\boldsymbol{U},\boldsymbol{X}_1^{N-1},\boldsymbol{Y}_1^{N-1}\right) \\ &\qquad \geq I\left(X_{2N};Y_{2N}|X_{1N},\boldsymbol{U},\boldsymbol{X}_1^{N-1},\boldsymbol{Y}_1^{N-1}\right). \end{aligned} \tag{116}$$

We proceed by induction. For $N = 1$, the inequality (94) reduces to (112) and is thus satisfied by Proposition 1. We next assume that (94) holds for $N - 1$

$$I\left(\boldsymbol{X}_2^{N-1};\boldsymbol{Y}_1^{N-1}|\boldsymbol{X}_1^{N-1},\boldsymbol{U}\right) \geq I\left(\boldsymbol{X}_2^{N-1};\boldsymbol{Y}_2^{N-1}|\boldsymbol{X}_1^{N-1},\boldsymbol{U}\right). \tag{117}$$

From (117) and Proposition 1 it follows that

$$I\left(\boldsymbol{X}_2^{N-1};\boldsymbol{Y}_1^{N-1}|\boldsymbol{X}_1^{N-1},\boldsymbol{U},X_{1N},Y_{2N}\right) \geq I\left(\boldsymbol{X}_2^{N-1};\boldsymbol{Y}_2^{N-1}|\boldsymbol{X}_1^{N-1},\boldsymbol{U},X_{1N},Y_{2N}\right) \quad (118)$$

where we let $V=(X_{1N},Y_{2N})$. The proof of Lemma 5 follows from (115), (116), and (118).

If the inputs X_1 and X_2 are independent, then the same steps as above apply, as long as $X_1 \rightarrow V \rightarrow X_2$ forms a Markov chain in Proposition 1 as in [13]. □

Proof of Theorem 7 (a): By sign reversal symmetry, it suffices to consider only the case $h_{21}>1$ and $h_{12}>0$. First we show that the GSI-UC conditions (102) and (103) imply (109). With nonnegativity of h_{21} and h_{12}, (102) directly implies

$$h_{12} \geq \frac{\alpha-1}{\alpha}+\frac{h_{21}}{\alpha}=1+\frac{h_{21}-1}{\alpha}. \quad (119)$$

Since $h_{21}>1$, it follows that $h_{12}\geq 1$. We now show that $h_{12}\alpha \geq 1$. For $\alpha \geq 1$, $h_{12}\geq 1$ implies $h_{12}\alpha\geq 1$. For $\alpha<1$, we suppose for the purpose of contradiction that $h_{12}\alpha<1$. In this case, $h_{21}>\alpha$ and (103) implies $1-h_{12}\alpha \geq h_{21}-\alpha$, or equivalently

$$\frac{1+\alpha}{\alpha}-\frac{h_{21}}{\alpha}\geq h_{12}. \quad (120)$$

Applying the upper bound in (120) to the lower bound in (119) yields the contradiction $h_{21}\leq 1$. It follows that $h_{12}\alpha\geq 1$ for all α.

To complete the forward proof, we observe that if $\alpha \geq 1$, then (109) follows directly from (119). On the other hand, if $\alpha<1$, then $\alpha<h_{21}$ and since $h_{12}\alpha\geq 1$, (103) implies $h_{12}\alpha-1\geq h_{21}-\alpha$, or equivalently

$$h_{12}\geq \frac{1-\alpha}{\alpha}+\frac{h_{21}}{\alpha}. \quad (121)$$

Since $\alpha<1$, (121) implies (109), completing the proof that the GSI-UC conditions imply (109).

For the reverse direction, we now show that (109) implies (102). We consider the cases $\alpha\geq 1$ and $\alpha<1$ separately. For $\alpha \geq 1$, (109) implies

$$h_{12}\alpha \geq \alpha-1+h_{21}. \quad (122)$$

From nonnegativity of h_{12}, α, and h_{21}, (102) follows. For $\alpha<1$, (109) implies

$$h_{12}\alpha\geq 1-\alpha+h_{21}. \quad (123)$$

Since $1>\alpha$, it follows that

$$h_{12}\alpha+1>1+h_{21}>\alpha+h_{21}. \quad (124)$$

From nonnegativity of h_{12}, α, and h_{21}, (102) follows.

Now we show (109) also implies (103). If $\alpha\geq h_{21}>1$ then (109) implies

$$h_{12}\alpha\geq \alpha-1+h_{21}>1. \quad (125)$$

Since $0>2-2h_{21}$, we have

$$h_{12}\alpha\geq\alpha-1+h_{21}+(2-2h_{21})=\alpha+1-h_{21}. \quad (126)$$

Since $h_{12}\alpha\geq 1$ and $\alpha\geq h_{21}$, (103) follows.

Next we assume that $h_{21}>\alpha\geq 1$. In this case, we observe that (109) still implies (125). Since $0\geq 2-2\alpha$, (125) implies

$$h_{12}\alpha\geq\alpha-1+h_{21}+(2-2\alpha)=1+h_{21}-\alpha. \quad (127)$$

Since $h_{12}\alpha\geq 1$ and $h_{21}>\alpha$, (103) follows.

Finally, we assume that $\alpha<1$. In this case, (109) implies $h_{12}\alpha\geq 1-\alpha+h_{21}$, or equivalently

$$h_{12}\alpha-1\geq h_{21}-\alpha. \quad (128)$$

As both the left and right side are nonnegative, (103) follows. □

Proof of Theorem 7 (b): By sign reversal symmetry, it suffices to consider only the opposite-sign case $h_{21}>1$ and $\hat{h}_{12}=-h_{12}>0$. We will show that the GSI-UC conditions (102) and (103), restated here in terms of $\hat{h}_{12}$ as

$$|\hat{h}_{12}\alpha-1|\geq|\alpha+h_{21}| \quad (129)$$

$$|\hat{h}_{12}\alpha+1|\geq|\alpha-h_{21}| \quad (130)$$

hold if and only if

$$\hat{h}_{12}\geq\frac{\alpha+1}{\alpha}+\frac{h_{21}}{\alpha}. \quad (131)$$

To show that the GSI-UC conditions imply (131) requires the following lemma.

Lemma 6: If $h_{21}>1$ and the GSI-UC conditions (129) and (130) hold, then $\hat{h}_{12}\alpha\geq 1$.

A proof of Lemma 6 follows below. From Lemma 6, (129) implies $\hat{h}_{12}\alpha-1\geq\alpha+h_{21}$, which is equivalent to (131).

For the reverse direction, we assume (131) is satisfied, implying

$$\hat{h}_{12}\alpha-1\geq\alpha+h_{21}. \quad (132)$$

Since $h_{21}>1$, both sides of (132) are nonnegative, and thus (129) holds. Furthermore, since $\hat{h}_{12}\alpha+1\geq\hat{h}_{12}\alpha-1$, (132) and the nonnegativity of α and h_{21} imply

$$\hat{h}_{12}\alpha+1\geq\alpha+h_{21}=|\alpha+h_{21}|\geq|\alpha-h_{21}|. \quad (133)$$

Thus, the GSI-UC condition (130) holds. □

Proof of Lemma 6: For the purpose of contradiction, we assume $\hat{h}_{12}\alpha<1$. Thus, by (129), we have $1-\hat{h}_{12}\alpha\geq\alpha+h_{21}$, or equivalently

$$1-\alpha-h_{21}\geq\hat{h}_{12}\alpha. \quad (134)$$

Now we must consider the cases $\alpha>h_{21}$ and $\alpha<h_{21}$ separately. If $\alpha>h_{21}$, then (130) implies

$$\hat{h}_{12}\alpha\geq\alpha-1-h_{21}. \quad (135)$$

Applying the upper bound in (134) to the lower bound in (135) yields $\alpha\leq 1$. In this case, this implies $h_{21}<\alpha<1$, which is a contradiction. If $\alpha\leq h_{21}$, then (130) implies

$$\hat{h}_{12}\alpha\geq h_{21}-\alpha-1. \quad (136)$$

Applying the upper bound in (134) to the lower bound in (136) yields $h_{21}\leq 1$, which is a contradiction. □

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